First Semester B.Sc. Degree Examinations

SEPTEMBER/OCTOBER 2022

(Semester Scheme) (New Syllabus 2018 Onwards)

MATHEMATICS

SSA 540: Paper – BSM I: Algebra – I and Calculus – I

Time: 3 hrs.] [Max. Marks: 70

Note: Answer ALL questions.

I. *Answer any FIVE of the following.* 5 x 2 = 10 Marks

1. If A and B are symmetric matrices of same order, then show that $A+B$ is also a symmetric matrix.

2. Find the value of λ for which the following system has a non – trivial solution.

$$7x+4y+3z=0$$
$$x+2y+\lambda z=0$$
$$x+3y+2z=0$$

3. If λ is an eigen value of the matrix A then prove that $\frac{1}{\lambda}|A|$ is an eigen value of adj.A provided A is non – singular.

4. Find the Angle between radius vector and Tangent for the curve $r=ae^{\theta.\cot\alpha}$

5. Find $\frac{ds}{dx}$ for the curve $y=a\log\sec\left(x/a\right)$

6. Find n^{th} derivative of e^{mx}.

7. If $x=r\cos\theta$ and $y=r\sin\theta$, then show that $\frac{\partial\theta}{\partial x}=\frac{-y}{x^2+y^2}$

8. If $u=x^2y+y^2z+z^2x$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=(x+y+z)^2$

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Prove that the rank of the transpose of a matrix is same as that of the original matrix.

2. Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$ using elementary row transformation.

3. For what values of λ, the system of equations

$$x+y+z=1$$
$$2x+y+4z=\lambda$$
$$4x+y+10z=\lambda^2$$

have a solution, solve completely.

4. Test the following system for consistency and solve if it is consistant.

$$2x+3y+2z=3$$
$$3y+z=0$$
$$x-2y=1$$

5. Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}-4 & -6\\ 3 & 5\end{bmatrix}$

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Diagonalise the Matrix $A=\begin{bmatrix}2 & 4\\ 0 & 5\end{bmatrix}$

2. Verify Cayley – Hamilton theorem for $A=\begin{bmatrix}2 & 1 & 0\\ 0 & 2 & 1\\ 1 & 1 & -1\end{bmatrix}$ and hence find A^{-1}.

3. Find the adjoint of a matrix $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 3 & 2\\ -1 & 1 & 2\end{bmatrix}$ using Cayley – Hamilton theorem.

4. Show that the Angle between radius vector and the tangent at a point on the curve $r=f(\theta)$ is $\tan\phi = \dfrac{r.\,d\theta}{dr}$

5. Show that the curves $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$ intersect orthogonally.

IV. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Find the pedal equation of the ellipse $x^2/a^2 + x^2/b^2 = 1$

2. Find the radius of curvature of the curve $y=4\sin x-\sin 2x$ at $x=\pi/2$

3. Find the co-ordinates of centre of curvature at (x,y) for the curve $y^2=4ax$.

Contd…………..3

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4. Find the n^{th} derivative of $y = \cos^4(x)$

5. Find n^{th} derivative of $e^{ax}.\sin(bx+c)$.

3 x 5 = 15 Marks

V. ***Answer* any *THREE of the following:***

1. State and prove Leibnitz theorem for the n^{th} derivative of product of two functions.
2. If $y = e^{m\sin^{-1}(x)}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.
3. If $u = \log(\tan x + \tan y + \tan z)$ then prove that $\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$.
4. Verify $\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial x}$ for the function $f = x\tan y + y\tan x$
5. Find the maximum and minimum values of the function $f(x, y) = x^3 + y^3 - 3xy$.

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First Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(Old Syllabus 2014 – 18)

MATHEMATICS

SSA 530: Paper–I

Time: 3hrs.] *[Max.Marks: 80*

Note: Answer ALL questions.

I. *Answer any TENof the following.* 10x2=20 Marks

1. If A is any matrix, then prove that $A'A$ and AA' are both symmetric matrices.
2. If x_1 and x_2 are two eigen vectors of Matrix A corresponding to same eigen value λ then show that $x_1 + x_2$ is also an eigen value for same λ.
3. Find the eigen values of the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$
4. Show that the multiplicative group of cube roots of unity is cyclic.
5. Find the number of generators of the cyclic group of order 18.
6. State Euler's theorem.
7. Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$
8. Define continuity of function.
9. Show that the function $f(x) = \begin{cases} x^2 & if \quad x \le 3 \\ 6x - 9 & if \quad x \ge 3 \end{cases}$ is differentiable at $x = 3$
10. Find the n^{th} derivative of $y = \cos(ax + b)$
11. Evaluate : $\int_0^{\pi} x \sin^8 x \, dx$
12. Evaluate : $\int \cot^6 x \, dx$

Contd.........2

II. *Answer any THREE of the following:* 3X5=15 Marks

1. Find the Rank of the matrix $\begin{bmatrix} 3 & -1 & -1 & 2 \\ 2 & 2 & 1 & -1 \\ 1 & -3 & 2 & 1 \\ 1 & -3 & 6 & -1 \end{bmatrix}$ using elementary row transformations.

2. Find the Inverse of the matrix $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by elementary row operations.

3. Solve the system of equations $\begin{aligned} x+2y+3z&=0 \\ 2x+3y+4z&=0 \\ 7x+13y+19z&=0 \end{aligned}$

4. Find the eigen values and theeigen vectors of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

5. Diagonalise the matrix $A=\begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$

III. *Answer any THREEof the following:* 3x5=15 Marks

1. Verify Caley – Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

2. Show that a non – empty subset H of group G is a subgroup of G iff $\forall a,b \in H,\ ab^{-1} \in H$

3. Prove that every subgroup of a cyclic group is cyclic.

4. State and prove Fermat's theorem.

5. Find the order of the permutation $f=\begin{pmatrix} 1 & 2 & 3 & 5 & 6 \\ 2 & 3 & 1 & 6 & 5 \end{pmatrix}$ Also verify wheather it is even or odd.

IV. *Answerany THREE of the following:* 3 x 5 = 15 Marks

1. If $f(x)=\begin{cases} x^2-1 & for \quad x<1 \\ 0 & for \quad x=1 \\ 1-\frac{1}{x} & for \quad x>1 \end{cases}$ show that the function is continuous at $x=1$.

Contd.........3

12. Find the centre and radius of the sphere $4x^2+4y^2+4z^2-16x-24y-12z+44=0$.

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. If $f:X\to Y$ be a function and A and B be two subsets of X. then prove that i) $A\subset B\Rightarrow f(A)\subset f(B)$ ii) $f(A\cup B)=f(A)\cup f(B)$

2. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=x^2$ and $g(x)=2x+5$ verify $(gof)^{-1}=f^{-1}og^{-1}$.

3. Find the equivalence relation determined by the partition P of the set $A=\{a,b,c,d\}$ where $P=\{(a,c)(b,d)\}$

4. Find the rank of the matrix $\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$

5. Find the inverse of the matrix $\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$ by elementary row operations.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Solve the system of equations
$2x-y+3z=0,\ 3x+2y+z=0,\ x-4y+5z=0$

2. Test the following system for consistency and solve if it is consistent.
$x+2y-z=3,\ 3x-y+2z=1,\ 2x-2y+3z=2$

3. Find the eigen value and eigen vectors of the matrix $\begin{bmatrix}-3 & 8\\ -2 & 7\end{bmatrix}$

4. Diagonalise the matrix $\begin{bmatrix}2 & 4\\ 0 & 5\end{bmatrix}$

5. Verify the Cayley – Hamilton theorem for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and hence find A^{-1}.

Contd.........6

IV. *Answerany THREE of the following:* 3 x 5 = 15 Marks

1. Discuss the continuity fo $f(x)=\begin{cases} x\sin\frac{1}{x}-1 & for \quad x\neq 0 \\ 0 & for \quad x=0 \end{cases}$ at $x=0$.

2. Examine the differentiability of $f(x)$ where $f(x)=\begin{cases} x^2-1 & if \quad x\geq 1 \\ 1-x & if \quad x<1 \end{cases}$ at $x=1$.

3. Find the n^{th} derivative of $e^{ax}\cos(bx+c)$.

4. If $y=a\cos(\log x)+b\sin(\log x)$ then prove that
$x^2y_{n+2}+(2n+1)x.y_{n+1}+(n^2+1)y_n=0$.

5. Find the n^{th} derivative of $x^2e^x\cos x$.

V. *Answer* any *THREE of the following:* 3 x 5 = 15 Marks

1. If $\vec{P}=(-2, 1)$, $\vec{Q}=(2, 5)$ and X lies on PQ. Xlies between Pand Q and $|\overline{QX}|=\frac{1}{3}|\overline{QP}|$.
Find co-ordinates of $\vec{X}$ in terms of $\vec{P}$ and $\vec{Q}$.

2. Find the equation of the plane passing through the point $(2, -1, 3)$, $(4, 0, 5)$ and $(2, 1, 7)$.

3. Determine whether the given planes are parallel if they are not parallel. Then find the parametric representation of the line of intersection of planes. $2x-6y+7z-5=0$ and $6x+y+4z-1=0$.

4. Determine the mutual position of the lines
$l_1: x=1-t,\ y=2+t,\ z=2t.$
$l_2: x=3-2s,\ y=4+2s,\ z=6+4s$

5. Find the intersection of the line $x=t,\ y=2t-1\ z=t+2$ with the sphere of radius $\sqrt{29}$ and centre at the origin.

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QP CODE 15122

First Semester B.Sc. Degree Examination

APRIL / MAY 2022

(Semester Scheme)

(Before 2014 – 15 Syllabus)

MATHEMATICS

SSA 530: Paper – I

Time: 3hrs.] *[Max. Marks: 80*

Note: Answer ALL questions.

I. *Answer any TEN of the following.* 10 x 2 = 20 Marks

1. Define equivalence relation with example.
2. Let $f:Q\rightarrow Q$ be defined by $f(x)=3x+1\,\forall x\in Q$ show that f is a bijection where Q is the set of all rational numbers.
3. If $f:R\rightarrow R$ is defined by $f(x)=2x+3$ and $g:R\rightarrow R$ is defined by $g(x)=3x-1$ find $fog(-1)$ and $gof(2)$..
4. Let A be any square matrix then prove that $A+A'$ is symmetric and $A-A'$ is skew symmetric matrix.
5. Express the matrix $\begin{bmatrix}1 & -1 & 3\\ 4 & 2 & 5\\ 6 & 0 & 7\end{bmatrix}$ as a sum of symmetric and skew symmetric matrices.
6. Find the Eigen values of the matrix $\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}$
7. Verify the continuity of the function $f(x)=\begin{cases}x^3 & when \quad x\le 2\\ x^2+4 & when \quad x>2\end{cases}$ at $x=2$..
8. Find the n^{th} derivative of $\dfrac{1}{ax+b}$.
9. Find the n^{th} derivative of a^{mx}.
10. Find the parametric equation of the line passing through the points $(3, 4, 5)$ and $(5, 4, 3)$.
11. Find the value of a if the planes $ax+2ay+10z-2=0$ and $x+2y+5z-7=0$ are parallel.

Contd.........5

2. If $f(x)=\begin{cases} 1+\sin x & for \quad 0<x<\frac{\pi}{2} \\ 2+\left(x-\frac{\pi}{2}\right)^2 & for \quad x\geq\frac{\pi}{2} \end{cases}$ at $x=\frac{\pi}{2}$. Discussthe continuity of the function.

3. If $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} g(x)=m$ then prove that $\lim_{x\to a}\left[f(x).g(x)\right]=lm$..

4. Prove that if a function is differentiable at point, then it is continuous at the point.

5. Examine the differentiability of the function $f(x)=\begin{cases} \dfrac{xe^{\frac{-1}{x}}-e^{\frac{1}{x}}}{e^{\frac{-1}{x}}+e^{\frac{1}{x}}} & if \quad x\neq 0 \\ 0 & if \quad x=0 \end{cases}$ at $x=0$.

V. *Answer* any *THREE of the following:* 3 x 5 = 15 Marks

1. Find the n^{th}derivative of x^m, where m is a positive integer.

2. If $y=a\cos(\log x)+b\sin(\log x)$, then show that $x^2 y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$

3. Obtain the reduction formula for $\int \cos^n x.\sin^m x\, dx$

4. Evaluate $\int_0^4 x^3\sqrt{4x-x^2}\, dx$

5. Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}\, dx$

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First Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme) (Old Syllabus 2018 Onwards)

MATHEMATICS

SSA 540: Paper – BSM I: Algebra – I and Calculus – I

Time: 3 hrs.] *[Max. Marks: 70*

Note: Answer ALL questions.

I. ***Answer any FIVE of the following.*** 5 x 2 = 10 Marks

1. If A is a square matrix, then prove that $A+A'$ is symmetric and $A-A'$ is skew symmetric matrix.

2. If λ is an eigen value of a square matrix A, then prove that λ^2 is an eigen value of A^2.

3. For what values of λ, the following system has non – trivial solution $x-3y+2z=0,\ 7x-21y+14z=0,\ -3x+9y-\lambda z=0$

4. Find angle between radius vector and tangent to the curve $r=a(1-\cos\theta)$

5. Find the radius of curvature of the curve $y=4\sin x-\sin 2x$ at $x=\pi/2$.

6. Find n^{th} derivative of e^{ax}.

7. If $x=r.\cos\theta$ and $y=r.\sin\theta$, show that $\frac{\partial r}{\partial x}=\frac{x}{\sqrt{x^2+y^2}}$ and $\frac{\partial r}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}$

8. Find stationary points of the function $f(x,y)=2x+4y-x^2-y^2-3$.

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Find the rank of the matrix $\begin{bmatrix}1&2&3&2\\2&3&5&1\\1&3&4&5\end{bmatrix}$ using elementary row transforms.

2. Find inverse of the matrix by using elementary row transformation $\begin{bmatrix}2&3&1\\1&2&3\\3&1&2\end{bmatrix}$

3. Solve the system of equations $x+2y-3z=0, 3x-y+z=0,\ 5x+3y+2z=0$

Contd............2

4. For what value of λ and μ, the following system $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ have a) No solution b) Unique solution c) An infinite solution.

5. Test the system for consistency and solve, $x+2y-z=1,\ 3x+8y+2z=28,\ 4x+9y-z=14$

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Find eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

2. Verify the Caylay – Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

3. Find inverse of the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ using Cayley – Hamilton theorem.

4. Find the value of ϕ for the curve $a\theta = \sqrt{r^2 - a^2} - a\cos^{-1}\left(a/r\right)$.

5. Show that the curves $r = ae^{\theta}$ and $re^{\theta} = b$ cut orthogonally.

IV. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Find pedal equation of the parabola $y^2 = 4a(x+a)$

2. Derive an expression for derivative of an arc length for the Cartesian curve $y = f(x)$

3. Find the co-ordinates of the centre of curvature at (x, y) for the curve $x^2 = 4ay$.

4. Find the n^{th} derivative of $\dfrac{x^2}{(x-1)^2(x-2)}$

5. Find n^{th} derivative of $\sin^2 x.\cos^3 x$

V. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. State and prove Leibnitz theorem for the n^{th} derivative of product of two functions.

2. If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} - n^2 y_n = 0$.

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3. If $f(x.y) = x^y + y^x$, then show that $\frac{\partial^2 f}{\partial x.\partial y} = \frac{\partial^2 f}{\partial y.\partial x}$

4. State and prove Euler's theorem for Homogeneous functions.

5. Find the maximum and minimum values of the function $f(x.y) = 2x^2 + 2xy + 2y^2 - 6x$.

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QP CODE 15122

First Semester B.Sc. Degree Examination

APRIL / MAY 2022

(Semester Scheme)

(Before 2014 – 15 Syllabus)

MATHEMATICS

SSA 530: Paper – I

Time: 3hrs.] [Max. Marks: 80

Note: Answer ALL questions.

I. *Answer any TEN of the following.* 10 x 2 = 20 Marks

1. Define equivalence relation with example.
2. Let $f: Q \to Q$ be defined by $f(x) = 3x+1 \forall x \in Q$ show that f is a bijection where Q is the set of all rational numbers.
3. If $f: R \to R$ is defined by $f(x) = 2x+3$ and $g: R \to R$ is defined by $g(x) = 3x-1$ find $fog(-1)$ and $gof(2)$..
4. Let A be any square matrix then prove that $A+A'$ is symmetric and $A-A'$ is skew symmetric matrix.
5. Express the matrix $\begin{bmatrix} 1 & -1 & 3 \\ 4 & 2 & 5 \\ 6 & 0 & 7 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrices.
6. Find the Eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$
7. Verify the continuity of the function $f(x) = \begin{cases} x^3 & when \quad x \le 2 \\ x^2+4 & when \quad x > 2 \end{cases}$ at $x = 2$..
8. Find the n^{th} derivative of $\dfrac{1}{ax+b}$.
9. Find the n^{th} derivative of a^{mx}.
10. Find the parametric equation of the line passing through the points $(3, 4, 5)$ and $(5, 4, 3)$.
11. Find the value of a if the planes $ax+2ay+10z-2=0$ and $x+2y+5z-7=0$ are parallel.

Contd………5

QP CODE 15122

2. If $f(x)=\begin{cases} 1+\sin x & \text{for } 0<x<\frac{\pi}{2} \\ 2+\left(x-\frac{\pi}{2}\right)^2 & \text{for } x\geq\frac{\pi}{2} \end{cases}$ at $x=\frac{\pi}{2}$. Discussthe continuity of the function.

3. If $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} g(x)=m$ then prove that $\lim_{x\to a}[f(x).g(x)]=lm.$.

4. Prove that if a function is differentiable at point, then it is continuous at the point.

5. Examine the differentiability of the function $f(x)=\begin{cases} \dfrac{xe^{\frac{-1}{x}}-e^{\frac{1}{x}}}{e^{\frac{-1}{x}}+e^{\frac{1}{x}}} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ at $x=0$.

V. *Answer* any *THREE of the following:* 3 x 5 = 15 Marks

1. Find the n^{th}derivative of x^m, where m is a positive integer.

2. If $y=a\cos(\log x)+b\sin(\log x)$, then show that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$

3. Obtain the reduction formula for $\int \cos^n x.\sin^m x\,dx$

4. Evaluate $\int_0^4 x^3\sqrt{4x-x^2}\,dx$

5. Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}\,dx$

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Contd.........4

Second Semester B.Sc., Degree Examinations
SEPTEMBER/OCTOBER 2022
(NEP Syllabus)

NSB 0240: ALGEBRA – II AND CALCULUS – II
Paper: MATHEMATICS – II

Time: 2 hrs] *[Max. Marks: 60*

SECTION – A

Answer all the questions:

1. ***Select the most appropriate answer from the options provided:*** $10 \times 1 = 10$

i) The set R – Q is called

a) Real number b) Rational number

c) Irrational number d) Complex number

ii) If B is countable subset of an uncountable set A, then $A \sim B$ is

a) Countable b) Uncountable

c) $A \cup B$ d) None of the above

iii) set A is closed if and only if its complement is

a) Open b) Closed

c) Not open d) None of the above

iv) The multiplicative inverse of i in the group of fourth roots of unity is

a) 1 b) – 1

c) i d) – i

v) If G is a finite group and H is any subgroup of G, then by Lagrange's theorem.

a) OH/OG b) OG/OH

c) $OH = OG$ d) $OH \neq OG$

type OH as $O(H)$ and OG as $O(G)$

Contd........... 2

vi) If $u = e^{x/y}$, then u_y becomes

a) $\frac{1}{y}e^{x/y}$

b) $\frac{1}{y^2}e^{x/y}$

c) $\frac{-x}{y^2}e^{x/y}$

d) $\frac{x}{y^2}e^{x/y}$

vii) For a continuous function $z = f(x,y)$, which of the following is true?

a) $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2}$

b) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$

c) $\frac{\partial^2 f}{\partial x^2} = -\frac{\partial^2 f}{\partial y^2}$

d) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

viii) The value of $\int \log x \, dx$ is

a) $x\log x - x + c$

b) $x\log x + x + c$

c) $x + \log x + c$

d) None

ix) The condition for $\int pdx + qdy$ to be path independent is

a) $\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$

b) $\frac{\partial p}{\partial x} = \frac{\partial q}{\partial y}$

c) $\frac{\partial p}{\partial x} = -\frac{\partial q}{\partial y}$

d) None

x) The value of $\int_0^1 \int_0^1 \int_0^1 dx\,dy\,dz$ is

a) 1

b) 2

c) 3

d) None

Contd............ 3

SECTION – B

Answer any FIVE of the following: $5 \times 3 = 15$

2. Show that there is no rational number whose square is 2.
3. Show that the union of an arbitrary family of open set is open.
4. Prove that every cyclic group is abelian.
5. Find the order of each elements of the group $G = \{1, -1, i, -i\}$ under multiplication.
6. Verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ for the function $f(x, y) = \tan^{-1}(x/y)$
7. If $x = r\cos\theta$ and $y = r\sin\theta$ then show that $\frac{\partial(x, y)}{\partial(r, \theta)} = r$
8. If C is a curve leading from the origin to $(1,1,1)$ then compute $\int_C 2xy\,dx + (x^2 + 2yz)\,dy + (y^2 + 1)\,dz$
9. Evaluate $\int_0^2 \int_0^1 \int_{-1}^1 (x^2 + y^2 + z^2)\,dx\,dy\,dz$

SECTION – C

Answer any THREE of the following: $3 \times 5 = 15$

10. Show that every subset of a countable set is countable.
11. Find all the right cosets of the subgroup $H = \{0, 3\}$ in the group $(Z_6, +_6)$
12. If H and K are any two subgroups of a group G, then prove that HK is a subgroup of G if and only if HK = KH.
13. If $u = xy(x + y)$ where $x = at^2$, $y = 2at$ then find $\frac{du}{dt}$.
14. Evaluate $\iint_R \frac{x^2 y^2}{x^2 + y^2}\,dx\,dy$ where 'R' is the annular region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$

Contd........... 4

SECTION – D

Answer the following: $2 \times 10 = 20$

15. a) Define order completeness for real numbers and prove that the set Q of rational numbers is not order-complete.

b) State and prove Archimedean property for real numbers. (6 + 4)

OR

c) Define centre of a group G and prove that it is a subgroup of G.

d) State and prove Fermat's theorem. (6 + 4)

16. a) State and prove Euler's theorem for homogeneous functions.

b) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3xy$ (6 + 4)

OR

c) Find the volume of the tetrahedron formed by the plane $x = 0, y = 0, z = 0$ and $6x + 4y + 3z = 12$

d) Show that $\int_0^{\pi/2} \frac{\log(1 + y\sin^2 x)}{\sin^2 x} dx = \pi\left(\sqrt{1+y} - 1\right)$ by applying differentiation under integral sign (where y is a parameter) (4 + 6)

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Third Semester B.Sc. Degree Examinations

SEPTEMBER/OCTOBER 2022

(Semester Scheme) (New Syllabus 2018 – 19 Onwards)

MATHEMATICS

SSC 540: Paper – BSM – III: ALGEBRA – III AND DIFFERENTIAL EQUATIONS – I

Time: 3 hrs.] *[Max. Marks: 70*

Note: Answer ALL questions.

I. ***Answer any FIVE of the following.*** 5 x 2 = 10 Marks

1. Define Normal subgroup with an example.
2. Show that every quotient group of an abelian group is an abelian group.
3. Verify whether the function $f:(R,+)\rightarrow(R,+)$ defined by $f(x)=5x+1$ is a homomorphism or not.
4. Find the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$
5. Solve : $(x^2+1)\frac{dy}{dx}=1$
6. Find the integrating factor of the differential equation. $x\frac{dy}{dx}-2y=2x$.
7. Find the singular solution of $y=px+\frac{a}{p}$
8. Solve $(D^3-13D+12)y=0$

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Prove that a subgroup H of a group G is normal if and only if every right coset of H in G is a left Coset of H in G.
2. Prove that the intersection of any two normal subgroups of a group is also a normal subgroup. Give an example to show "The Union of two normal subgroups of a group is need not be a normal subgroup".
3. Show that every factor group of a cyclic group is cyclic.

Contd...............2

4. Let $f: G \to G'$ be a homomorphism from the group G into G' with Kernel K. Then prove that f is one – one if and only if $K = \{e\}$. Where 'e' is the identity element of G.

5. Let R^+ be the multiplicative group of all positive real numbers and R be the additive group of all real numbers. Then show that the mapping $f: R^+ \to R$ defined by $f(x) = \log x \, \forall \, x \in R^+$ is an isomorphism.

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Solve: $\frac{dy}{dx} = \frac{6x-2y-7}{3x-y+4}$

2. Solve: $(x^2+2y^2)dx - xydy = 0$

3. Solve: $\sin x \cos x \frac{dy}{dx} = y + \sin x$

4. Solve: $x.\frac{dy}{dx} + (1-x)y = x^2y^2$

5. Verify the exactness and solve $(4x+3y+1)dx + (3x+2y+1)dy = 0$

IV. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Solve : $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ by finding the integrating factor.

2. Solve : $xp^2 + (y-x)p - y = 0$

3. Solve : $y - 2px + yp^2 = 0$

4. Find the general and singular solution of $(x^2-1)p^2 - 2xyp + y^2 - 1 = 0$

5. Show that the family of curves $y^2 = 4a(x+a)$ is self orthogonal.

V. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Solve : $(D^3 - 3D^2 + 4D - 2)y = e^x$.

2. Solve : $(D^2 - 2D + 5)y = \sin 3x$

Contd...............3

3. Solve : $y'' - 2y' + y = x \cos x$

4. Solve : $4x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} - y = 4x^2$

5. Solve $\frac{dx}{dt} - 7x + y = 0$

$$\frac{dy}{dt} - 2x - 5y = 0$$

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Third Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme) (New Syllabus 2018 Onwards)

MATHEMATICS

SSC 540: Paper – BSM 3: ALGEBRA – III AND DIFFERENTIAL EQUATIONS – I

Time: 3 hrs.] [Max. Marks: 70

Note: Answer ALL questions.

I. *Answer any FIVE of the following.* 5 x 2 = 10 Marks

1. Let H be a subgroup and K be a normal subgroup of the group G. Then show that $H \cap K$ is normal in H.
2. Show that every quotient group of an abelian group is abelian.
3. Let G be a group of order 2P, where 'P' is prime then show that G has a normal subgroup of order 2P.
4. Solve $\frac{dy}{dx}+\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}=0$
5. Solve $\left(e^y+1\right)\cos x\,dx+e^y\sin x\,dy=0$
6. Solve $y=p\sin p+\cos p$
7. Find the general and singular solution of $y=px+\frac{a}{p}$
8. Solve $\left(D^3-3D^2+4\right)y=0$

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Show that A sub group H of a group G is normal iff $xHx^{-1}=H\ \forall x\in G$
2. Show that every quotient group of a cyclic group is cyclic.
3. If f is a homomorphism of group G into G' with kernel k then k is a normal subgroup of G.
4. Show that the additive group $(R,+)$ of all real numbers and the multiplicative group $(R^+,\cdot)$ of all positive real numbers are isomorphic under the mapping defined by $f(x)=e^x,\ \forall x\in R$

Contd...............2

5. Define $f: Z \to H$ by $f(x) = \begin{cases} 1 & \text{if } x \text{ is even} \\ -1 & \text{if } x \text{ is odd} \end{cases}$ show that f is homomorphism.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Solve: $\frac{dy}{dx} = \frac{1}{\cos(x+y)}$

2. Solve: $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)dx + x\sec^2\left(\frac{y}{x}\right)dy = 0$

3. Solve: $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

4. Solve: $\frac{x.dy}{dx} + y = y^2.\log x$

5. Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

IV. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Find the general and singular solution of the equation $(px-y)(py+x) = p$ using the substitution $x^2 = u$ and $y^2 = v$.

2. Solve : $(p-x)(p-y)(p+x+y) = 0$

3. Solve : $(x^2p^2 - 2xpy + y^2) = p^2 + 1$

4. Find the orthogonal trajectories of the family of the curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ where λ is a parameter.

5. Find orthogonal trajectories of family of curves $r = a.\cos^2 \theta/2$

V. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Solve : $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x \sin x$

2. Solve : $(D^3 - 3D - 2)y = x^2$

Contd..............3

3. Solve : $(D^2 - 3D + 2)\,y = \cosh x$

4. Solve $4x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} - y = 4x^2$

5. Solve $\dfrac{dx}{dt} + \dfrac{dy}{dt} - 2y = 2\cos t - 7\sin t$ and $\dfrac{dx}{dt} - \dfrac{dy}{dt} + 2x = 4\cos t - 3\sin t$

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Third Year B.Sc., Degree Examination

NOVEMBER/DECEMBER 2022

(Directorate of Distance Education)

Paper IV: DSC 231: MATHEMATICS

Time: 3 hrs] *[Max. Marks: 90*

Instructions:

Answer any SIX of the following:

SECTION – A

1) a) i) Evaluate $\int_C (3x+y)\,dx+(2y-x)\,dy+z^2dz$ where 'C' is the curve and $x=3t, y=t, z=2t$ and $0\le t\le 1$

ii) Evaluate $\int_0^1\int_0^{x^2}(x^2+y^2)\,dx\,dy$ (2 + 2)

b) Evaluate $\int_C\left(\frac{a^2y^2}{b^2}+\frac{b^2x^2}{a^2}\right)^{\frac{1}{2}}ds$ where 'C' is a ellipse whose equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(5)

c) Evaluate $\iint_D xy\left(x^2+y^2\right)^{\frac{3}{2}}dx\,dy$ where D is the positive quadrant of the unit circle.

(6)

2) a) i) Evaluate $\int_0^1\int_0^1\int_0^y xyz\;dx\;dy\;dz$

ii) Evaluate $\int_0^a\int_0^a\int_0^a(yz+zx+xy)\,dx\,dy\,dz$ (2 + 2)

b) Find the surface area of the portion of the plane $3x+2y+6z=12$ inside the cylinder $x^2+y^2=1$.

(5)

c) Evaluate $\iiint_V\left(x^2+y^2+z^2\right)dx\,dy\,dz$ where V is bounded by $x+y+z=a$ and $x=0, y=0, z=0$. (6)

Contd........... 2

SECTION – B

3) a) i) Prove that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta.\cos^{2n-1}\theta\, d\theta$

ii) Show that $\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$ (2 + 2)

b) Prove that $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = \beta(m,n)$ (5)

c) Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}}\,dx$ (6)

4) a) i) If $f(x)$ is R – integrable over $[a,b]$ and A is a constant then prove that

$$\int_a^b A.f(x)\,dx = A.\int_a^b f(x)\,dx$$

ii) Let f be a function defined on $[0,1]$ by $f(x) = \begin{cases} 0, & when \quad x \text{ is irrational} \\ 1, & when \quad x \text{ is rational} \end{cases}$

show that f is not R – integrable over $[0,1]$. (2 + 2)

b) If $f(x) = x^2$ on $[0,a], a > 0$ then show that $f \in R[0,a]$, and also show that $\int_0^a f(x)\,dx = \frac{a^3}{3}$. (5)

c) Evaluate $\int_{-2}^{2} x.|x|\,dx$ using integral as a limit of sum. (6)

SECTION – C

5) a) i) Find the part of complimentary function of $y'' + (1-\cot x)y' - y\cot x = \sin^2 x$

ii) Find the Wranskian of the equation $(D^2+1)y = \tan x$ (2 + 2)

b) Solve by changing the independent variable

$\frac{d^2y}{dx^2} - (1+4e^x)\frac{dy}{dx} + 3e^{2x}y = e^{2(x+e^x)}\ (x > 0)$ using the transformation $z = x^2$. (5)

c) Solve by changing the dependent variable

$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-1)y = -3e^{x^2}\sin 2x\ (x > 0)$ by removing the first derivative. (6)

6) a) i) Verify the condition of exactness of the equation

$$\cos x\frac{d^2y}{dx}+2\sin x\frac{dy}{dx}+3\cos x.\, y=\tan^2 x$$

ii) Verify the condition for integrability of $yz\log z\,dx-zx\log z dy+xydz=0$

(2 + 2)

b) Solve $\dfrac{dx}{z(x+y)}=\dfrac{dy}{z(x-y)}=\dfrac{dz}{x^2+y^2}$ (5)

c) Solve $(x^2+1)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=6(x^2+1)^2$, given $y=-2$, when $x=-2$ and $y=4$ when $x=2$ by the method of variation of parameters. (6)

SECTION – D

7) a) i) Solve $p^2+q^2=4$

ii) Solve $p^2+q^2=x+y$ (2 + 2)

b) Solve $z^2(p^2+q^2)=x^2+y^2$ (5)

c) Solve $(p^2+q^2)y=qz$ by using Charpits method. (6)

8) a) i) Find the half range sine series of the function $f(x)=\pi-x$ in $0<x<\pi$.

ii) Find the Fourier constant b_n for the function $f(x)=e^x$ in $-\pi<x<\pi$.

(2 + 2)

b) Find the Fourier series expansion with period '3' to represent the function $f(x)=2x-x^2$ in the range $(0,3)$ (5)

c) Expand $f(x)=x^2$ as a Fourier series in the interval $(-\pi,\pi)$ and hence show that $\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\ldots\ldots\ldots\ldots=\dfrac{\pi^2}{6}$ (6)

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Third Year B.Sc., Degree Examination

NOVEMBER/DECEMBER 2022

(Directorate of Distance Education)

Paper V: DSC 232: MATHEMATICS

Time: 3 hrs] *[Max. Marks: 90*

Instructions:

1) *Answer any SIX of the following:*
2) *Scientific Calculator is allowed*

SECTION – A

1) a) i) Show that $amp(z-2)=\frac{\pi}{2}$ represents a line parallel to imaginary axis.

ii) Evaluate $\lim_{z\to(1+i)}\left(\frac{z^2-z+1-i}{z^2-2z+2}\right)$ (2 + 2)

b) Find the equation of circle passing through the points 1, i and $1+i$. Also find its centre and radius. (5)

c) Find whether the function $f(z)=\frac{z+i}{z-i}$ is differentiable at $z=1+i$. (6)

2) a) i) Find the real and imaginary parts of $\log z$

ii) Show that $u=(x-1)^3-3xy^2+3y^2$ satfies Laplace equation. (2 + 2)

b) State and prove sufficient condition for the function $f(z)=u+iv$ is to be analytic. (5)

c) If $f(z)=u+iv$ is analytic then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}|f(z)|\right]^2=|f'(z)|^2$ (6)

SECTION – B

3) a) i) Evaluate $\int_C (\bar{z})^2 dz$ around the circle $|z|=1$

ii) Find the fixed points of the Bilinear transformation $w=\frac{i-z}{z+i}$ (2 + 2)

b) Evaluate $\int_C \frac{z}{(z^2+1)(z^2-9)} dz$, where 'C' is the circle $|z|=2$ (5)

c) Prove that Bilinear transformation preserves the cross ratio of four points. (6)

4) a) i) Evaluate $\Delta \tan^{-1} ax$

ii) Show that $\Delta - \nabla = \Delta.\nabla$ (2 + 2)

b) Using Newton's divided difference formula. Find the values of $f(18)$ and $f(15)$ from the following table. (5)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

c) Find $f'(2.2)$ and $f''(2.2)$ from the following table. (6)

x	1.4	1.6	1.8	2.0	2.2
$f(x)$	4.0552	4.9530	6.0496	7.3891	9.0250

SECTION – C

5) a) i) Find $L[\sin^2 t]$

ii) Find $L[\cosh 4t.\sin 3t]$ (2 + 2)

b) Find the Laplace transform of the function $f(t)$ with period $\frac{2\pi}{w}$. Where

$$f(t)=\begin{cases} \sin wt & 0<t\le\frac{\pi}{w} \\ 0 & \pi/_w \le t \le \frac{2\pi}{w} \end{cases}$$ and $f(t+7)=f(t)$. (5)

c) Express the function $f(t)$ in terms of unit step function and find its Laplace transformation where $f(t)=\begin{cases} t^2 & 0<t<2 \\ 6 & t>2 \end{cases}$ (6)

Contd............ 3

6) a) i) Find $L^{-1}\left[\frac{e^{-3s}}{(s+1)^3}\right]$

ii) Find $L^{-1}\left[\frac{s}{(s+4)^2}\right]$

(2 + 2)

b) Solve the integral equation $f(t) = at + \int_0^t f(u)\sin(t-u)\,du$ (5)

c) Solve $y'' + 2y' + 5y = e^{-t}.\sin t$ given $y(0) = 0$ and $y'(0) = 1$ (6)

SECTION – D

7) a) i) Evaluate $\Delta^{10}(1-x)(1-2x^2)(1-3x^3)(1-4x^4)$ for $h = 1$

ii) If $u_0 = 3,\ u_1 = 12,\ u_2 = 81,\ u_3 = 200,\ u_4 = 100,\ u_5 = 8$. find the value of $\Delta^5 u_0$

(2 + 2)

b) Using Simpson's $3/8^{th}$ rule obtain an approximate value of $\int_0^{0.3} (2x - x^2)^{1/2}\,dx$ with $h = 0.05$ (5)

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using the Weddle's Rule and Simpson's $1/3$ rule with $h = 1$.

(6)

8) a) i) Evaluate $\int_0^1 e^x dx$ approximately in steps of 0.2 using the Trapezodial rule.

ii) Using Euler's method. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$. Given $y = 1$ when $x = 0$. Find y for $x = 0(0.1)0.2$

(2 + 2)

b) Solve $\frac{dy}{dx} = 1 + xy$ using Picard's method of successive approximation to find $y^{(3)}(0.2)$. Given $y = 0$ when $x = 0$. (5)

c) Solve $\frac{dy}{dx} = x + y^2$ with initial condition $y = 1$ when $x = 0$ for $x = 0(0.2)0.4$ using Runge – Kutta fourth order method. (6)

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Third Year B.Sc., Degree Examination

NOVEMBER/DECEMBER 2022

(Directorate of Distance Education)

Paper III: DSC 230: MATHEMATICS

Time: 3 hrs] *[Max. Marks: 90*

Instructions:

Answer any SIX of the following:

SECTION – A

1) a) i) Define normal subgroup of a group.

ii) Prove that the intersection of two normal subgroups of a group is also a normal subgroup. (2 + 2)

b) Prove that the product of any two normal subgroups of a group is a subgroup of the group. (5)

c) Let $f: G \to G'$ is a homoorphism of groups with Kernel K. Then prove that f is one – one if and only if $K = \{e\}$, where 'e' is identity of G. (6)

2) a) i) Define commutative ring with unity give one example.

ii) Prove that in an integral domain, the non zero elements form a semi group with respect to multiplication. (2 + 2)

b) Prove that every left ideal of a ring R is a subring of R and disprove the converse part. (5)

c) Prove that a commulative Ring 'R' with unity is a Field if and only if R has no proper ideals. (6)

SECTION – B

3) a) i) Define a polynomial ring and give one example.

ii) If $f(x) = 2 - 5x^2 + 7x^3$ and $g(x) = 3 + 4x - x^2 + 8x^3 + 9x^4$ find the degree of $f(x) + g(x)$ (2 + 2)

b) State and prove Factor theorem. (5)

c) Decompose the polynormal $x^4 + x^3 + x + 1$ over z_3. (6)

Contd........... 2

4) a) i) Let $(Z,+)$ be the group of integers under addition. If $f: Z \to Z$ defined by $f(x) = x+1 \forall x \in Z$, Verify f is a homomorphism or not.

ii) Show that $\phi: Z \to 3Z$ such that $\phi(n) = 3n : \forall n \in Z$ is an isomorphism from the additive group of integers into the additive group of integral multiples of 3. Find its Kernel. (2 + 2)

b) Let $f: G \to G'$ be a homorphism of groups with kernel K. Then prove that K is a normal subgroup of G. (5)

c) Let $G = \{x + y\sqrt{2} / x, y \in Z\}$. Define $f:(G,+) \to (G,+)$ by $f:(x+y\sqrt{2}) = x - y\sqrt{2}$ Show that f is a homomorphism and find its kernel. Verify whether f is an isomorphism. (6)

SECTION – C

5) a) i) If $V(F)$ is a vector space then prove that $a\alpha = a\beta$ and $a \neq 0 \Rightarrow \alpha = \beta \forall a \in F \,\&\, \alpha, \beta, \in V$

ii) Show that $W = \{(x,y,z)/3x_1, -x_2 + x_3 = 0\}$ is a subspace of $V_3(R)$ (2 + 2)

b) Show that the vector $(3,-7,6)$ is in the span of the vectors $(1,-3,2),(2,4,1)$ and $(1,1,1)$ (5)

c) Prove that the set $S = \{\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots\alpha_n\}$ of non zero vectors are linearly Dependent If and only if some vector $\alpha_k\,(K \geq 2)$ is a linear combination of its preceeding vectors. (6)

6) a) i) Show that the set $S = \{(1,1,-1),(2,-3,5),(-2,1,4)\}$ is linearly independent in $V_3(R)$

ii) Prove that the super set of a linearly dependent set is linearly dependent. (2 + 2)

b) If V is a vector space of dimension 'n' then prove that (a) Any $(n+1)$ vectors of V are linearly dependent (b) No set of $(n-1)$ vectors can span V. (5)

c) Find the basis and dimension of the sub space spanned by $S = \{(1,1,1)(2,1,2)(1,0,1)(5,3,5)\}$ in $V_3(R)$. (6)

Contd........... 3

SECTION – D

7) a) i) If $T:V \to W$ is a linear Transformation then prove that $T(0)=0'$, where 0 and $0'$ are the zero vectors of V and W respectively.

ii) Find a linear Transformation $T:V_2(R)\to V_2(R)$ defined by $T(1,0)=(1,-1)$ and $T(0,1)=(0,-2)$ (2 + 2)

b) Find the matrix of the linear transformation $T:V_2(R)\to V_2(R)$ defined by $T(2,3)=(1,0)$, $T(3,2)=(1,-1)$ (5)

c) Let $T:V \to W$ is a Linear Transformation prove that (a) $R(T)$ is a subspace of W. (b) $N(T)$ is a subspace of V. (6)

8) a) i) If $Z=\sqrt{x^2+y^2}$ find $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}$

ii) If $u=x^y$ prove that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$ (2 + 2)

b) If $z=\log\sqrt{x^2+y^2}$ then prove that $x^3z_x-y^3z_y=x^2-y^2$ (5)

c) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that $xu_x+yu_y=\frac{1}{2}\tan u$ (6)

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Fourth Semester B.Sc. Degree Examinations

SEPTEMBER/OCTOBER 2022

(Semester Scheme) (New Syllabus 2018 – 19 Onwards)

MATHEMATICS

SSD 540: Paper – BSM 4: DIFFERENTIAL EQUATIONS – II AND ANALYSIS

Time: 3 hrs.] [Max. Marks: 70

Note: Answer ALL questions.

I. *Answer any FIVE of the following.* 5 x 2 = 10 Marks

1. Find the part of the complementary function $y'' - \cot x . y' - (1 - \cot x) y = e^x \sin x$.
2. Verify whether the differential equation $x^2(1+x)\dfrac{d^2y}{dx^2} + 2x(2+3x)\dfrac{dy}{dx} + 2(1+3x)y = 0$ is exact or not
3. Find Wronskian of the differential equation $\dfrac{d^2y}{dx^2} + 4y = 4\tan 2x$
4. Find the normal form of the differential equation $\dfrac{d^2y}{dx^2} + p\dfrac{dy}{dx} + qy = R$
5. Prove that every convergent sequence is bounded.
6. Examine the behavior of the sequence $\{a_n\}$ where $a_n = \sqrt{n}\left(\sqrt{n+2} - \sqrt{n+1}\right)$
7. Discuss the convergence of the series. $\dfrac{1}{1.4} + \dfrac{1}{2.5} + \dfrac{1}{3.6} + \dots\dots\dots\dots$
8. Test the convergence of the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2+1}$ using integral test.

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Solve $x^2y'' - x(x+1)y' + 2(x+1)y = x^3 \ (x > 0)$ by finding a part of complementary function.
2. Solve $\dfrac{d^2y}{dx^2} + \tan x\dfrac{dy}{dx} + 4y\cos^3 x = 0$ by changing independent variable.

Contd...............2

3. Solve $\left(x^2+1\right)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+\left(9x^2+11\right)y=0$ by changing dependent variable.

4. Solve $x^2y_2+xy_1-y=x^2e^x\left(x>0\right)$ by the method of variation of parameters.

5. Show that the equation $\left(2x^2+3x\right)\frac{d^2y}{dx^2}+\left(6x+3\right)\frac{dy}{dx}+2y=\left(x+1\right)e^x$ is exact and solve.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Solve $y''+y'\cot x-y\,\mathrm{cosec}^2x=0$ given that $\cot x$ is a part of complementary function.

2. Solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$ by the method of variation of parameters.

3. If $\{x_n\}$ and $\{y_n\}$ are the two sequences such that $\lim_{n\to\infty}x_n=l$ and $\lim_{n\to\infty}y_n=m$ then show that $\lim_{n\to\infty}\left(x_n+y_n\right)=l+m$.

4. Discuss the behaviour of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$.

5. Show that the sequence $\{x_n\}$ defined by $x_n=\frac{1}{n+1}+\frac{1}{n+2}+..........+\frac{1}{n+n}$ is convergent.

IV. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Show that the sequence $\{x_n\}$ where $x_n=\frac{3n-4}{4n+3}$ is i) monotonic increasing ii) bounded iii) converges to $\frac{3}{4}$.

2. Discuss the convergence of the sequence $\left\{n^{\frac{1}{n}}\right\}$

3. Show that the sequence $\{x_n\}$ defined by $x_1=1$ and $x_{n+1}=\sqrt{7x_n}$ converges to 7.

4. Let Σu_n and Σv_n be two series of positive terms and $\lim_{n\to\infty}\frac{u_n}{v_n}$ be finite and non zero. Show that Σu_n and Σv_n converge or diverge together.

5. Discuss the convergence of the series $\sum\left(\sqrt{n^2+1}-\sqrt{n^2-1}\right)$

V. *Answer* **any** ***THREE of the following:*** 3 x 5 = 15 Marks

1. Show that $\sum\frac{1}{n^p}$ is convergent if $p>1$ and divergent if $p\le 1$.

2. Test the convergence of the series $\frac{x}{1.3}+\frac{x^2}{3.5}+\frac{x^3}{5.7}+\ldots\ldots\ldots$

3. State and prove Raabe's test.

4. Discuss the convergence of the series $\sum\frac{4.7.10.\ldots\ldots\ldots\ldots(3n+1)}{1.2.\ldots\ldots\ldots\ldots\ldots\ldots\ldots n}x^n$

5. Examine the convergence of the series $\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{n(n-1)}$, $0<x<1$.

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Fourth Semester B.Sc. Degree Examination

APRIL / MAY 2022

(Semester Scheme)

(Old Syllabus 2014 – 18)

MATHEMATICS

SSD 530: Paper – IV

Time: 3hrs.] *[Max. Marks: 80*

Note: Answer ALL questions.

I. ***Answer any TEN of the following.*** 10 x 2 = 20 Marks

1. Find the part of Complementary function of $x^2y''+xy'-y=2x^2$
2. Find the Wronskian's for the equation $\frac{d^2y}{dx^2}+y=\tan x$
3. Show that the equation $(1+x^2)y''+4xy'+2y=\sec^2 x$ is exact.
4. Solve $\frac{dx}{x}=\frac{dy}{-y}=\frac{dz}{y^2-x^2}$
5. Form the partial differential equation of $z=(x-a)^2+(y-b)^2$ by removing the arbitrary constants. a and b.
6. Solve $p^2+q^2=1$
7. Define Divergent sequence and give an example.
8. By using the definition of the limit of a sequence show that $\lim_{n\to\infty}\frac{2n+1}{n+3}=2$
9. Examine the behaviour of the sequence whose n^{th} term is $\frac{(n+1)^{n+1}}{n^n}$
10. If a series $\sum_{n=1}^{\infty}u_n$ is convergent, then prove that $\lim_{n\to\infty}u_n=0$.
11. Test the convergence of the series $\sum\left(\frac{n}{n+1}\right)^{n^2}$
12. Sum to infinity the series $1+\frac{1}{3}.\frac{1}{3^2}+\frac{1}{5}.\frac{1}{3^4}+$..........

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Solve $y''+y'\cot x - y\operatorname{cosec}^2 x = 0$ given that $\cot x$ is a part of complimentary function.

2. Solve $(1+x^2)^2\dfrac{d^2y}{dx^2}+2x(1+x^2)\dfrac{dy}{dx}+y=0$ using the transformation $Z=\tan^{-1}x$..

3. Solve $y''-2\tan x\, y'+5y=\sec x\, e^x$ by the method of changing the dependent variable.

4. Solve $(2x^2+3x)y''+(6x+3)y'+2y=(x+1)e^x$ by verifying it for exactness.

5. Solve $x^2y''-4xy'+6y=x^4\sin x$ by the method of variation of parameters.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Verify the condition for integrability and solve
   $(yz+2x)dx+(zx-2z)dy+(xy-2y)dz=0$

2. Solve $\dfrac{dx}{x^2-y^2-z^2}=\dfrac{dy}{2xy}=\dfrac{dz}{2xz}$

3. Find the general and singular solutions of $z=px+qy+2\sqrt{pq}$

4. Solve $z^2(p^2+q^2+1)=1$

5. Solve $z=px+qy+p^2+q^2$ by Charpits method.

IV. *Answerany THREE of the following:* 3 x 5 = 15 Marks

1. Prove that every convergent sequence is bounded.

2. If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty}(a_n-b_n)=a-b$

3. Discuss the convergence of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$

4. Show that the sequence $\left[\dfrac{3n+4}{2n+1}\right]$ is
   a) Monotonically decreasing
   b) Bounded
   c) Tend to the limit $\frac{3}{2}$.

5. Find the nature of the series
   $\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+........$

Contd........ 3

2. Show that finite integral domain is a field.
3. Show that a subset S of R is a subring iff

   $(i)\ \forall a,b\in S,\ a-b\in S$ $(ii)\ \forall a,b\in S,\ a\bullet b\in S.$
4. Prove that an arbitrary intersection of left ideals of a ring is again a left ideal of the ring.
5. Find the principal and Maximal ideals of Z_{12}.

III. ***Answer any THREEof the following:*** 3x5=15 Marks

1. Show that the ring $R=\{a+b\sqrt{2},a,b\in z\}$ and $S=\{a+b\sqrt{3}/a,b\in z\}$ are not isomorphic.
2. State and prove fundamental theorem of homomorphism.
3. If $f(x)=x^4-3x^3+2x^2+4x-1$ and $g(x)=x^2-2x+3$ over the range Z_5. Find $q(x)$ and $r(x)$.
4. Test the rational root of the polynomial $3x^3+x^2+x-2$.
5. If $p(x)$ is irreducible $p(x)$ then divides $a(x).b(x)$.

IV. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. Prove that limit of a convergent sequence is unique.
2. If $\lim_{n\to\infty} S_n=a$ and $\lim_{n\to\infty} T_n=b$, , where $b\neq o, T_n\neq 0\ \forall n$ then show that $\lim_{n\to\infty}\frac{S_n}{T_n}=\frac{a}{b}$.
3. Examine the convergence of the sequence $u_n=\frac{1^2+2^2+3^2+........+n^2}{1+2+3+..........+n}$.
4. Show that the sequence $\{u_n\}$ where n^{th} term is given by $u_n=\frac{3n+4}{2n+1}$ is

   (i) Monotonically increasing (ii) bounded (iii) tend to the limit $3/2$.
5. Show that sequence $\{u_n\}$ defined by $u_1=\sqrt{2}$ and $u_{n+1}=\sqrt{2.u_n}$ converges to 2.

V. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. Discuss the convergence of the series $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+............+.......$

Contd.........6

2. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{4.7.10.........(3n+1)}{1.2.3............n} . x^n$.

3. State and prove Cauchy's root test.

4. Find the sum to infinity of the series $\frac{2}{2!}\left(\frac{1}{12}\right)^2 + \frac{2.5}{3!}\left(\frac{1}{12}\right)^3 + \frac{2.5.8}{4!}\left(\frac{1}{12}\right)^4 + ...$

5. Find the sum to infinity of the series $S = \frac{1}{1.3}\left(\frac{1}{2}\right) + \frac{1}{2.4}\left(\frac{1}{2}\right)^2 + ...$

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QP CODE 15422

Fourth Semester B.Sc. Degree Examination

APRIL / MAY 2022

(Semester Scheme)

(Before 2014 – 15 Old Syllabus)

MATHEMATICS

SSD 530: Paper–IV

Time: 3hrs.] [Max.Marks: 80

Note: Answer ALL questions.

I. *Answer any TENof the following.* 10x2=20 Marks

1. Define integral domain with an example.
2. In a ring $(R,+,\bullet)$, Show that $(-a)(-b)=a\bullet b, \forall\ a,b\in R$..
3. Define subring of a ring with an example.
4. Show that every subring of R need not be an ideal of R.
5. Show that $f:Z\to 2Z$ define by $f(x)=2x$ is not a homomorphism.
6. Find the associates in Z_7.
7. Test the convergence of the sequence $x_n=\dfrac{n+(-1)^n}{n}$.
8. Show that the sequence $x_n=3+\dfrac{1}{n}$ is monotonic.
9. Define bounded sequence and give an example.
10. Show that if $\sum u_n$ converges then $\lim\limits_{n\to\infty} u_n=0$.
11. Find the nature of the series $\sum \tan\dfrac{1}{n}$.
12. Test for the absolute convergence of the series $1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+$.........

II. *Answer any THREE of the following:* 3X5=15 Marks

1. In the set of integers $*$ and $\bullet$ are defined $a*b=a+b-1$ and $a\bullet b=a+b-ab, \forall a,b\in R$. Prove that $(z,*,\bullet)$ is a commutative ring with unity. Is it an integral domain?

Contd.........5

QP CODE 15422

V. *AnsweranyTHREE of the following:* 3 x 5 = 15 Marks

1. State and prove D'Alembert's ratio test.

2. Examine the convergence of the series $\frac{x}{1^3}+\frac{x^2}{2^3}+\frac{x^3}{3^3}+\ldots\ldots\ldots$

3. Test the convergence of the series $\sum\frac{[1+nx]^n}{n^n}$

4. Sum to infinity the series $1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+\ldots\ldots$

5. Sum to infinity the series $\frac{1^2}{1!}+\frac{2^2}{2!}+\frac{3^2}{3!}+\ldots\ldots\ldots$

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Contd.........4

Fourth Semester B.Sc. Degree Examination

APRIL / MAY 2022

(Semester Scheme) (New Syllabus 2018 Onwards)

MATHEMATICS

SSD 540: Paper – BSM 4: DIFFERENTIAL EQUATIONS – II AND ANALYSIS

Time: 3 hrs.] [Max. Marks: 70

Note: Answer ALL questions.

I. *Answer any FIVE of the following.* 5 x 2 = 10 Marks

1. Find the part of complementary function of the differential equation
$$x\frac{d^2y}{dx^2}-(2x+1)\frac{dy}{dx}+(x+1)y=x^2e^x.$$

2. Verify the differential equation is exact or not $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0\ (x\neq 1)$

3. Find the Wronskian of the equation $\frac{d^2y}{dx^2}+y=\tan x$

4. Write the normal form of the equation $\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy=R$

5. Define convergent sequence with an example.

6. Find the supremum and infimum of the sequence $\left\{\frac{1}{n}\right\}$.

7. Show that the series $1+2+3+\ldots\ldots\ldots+n+\ldots\ldots\ldots$ diverges to $+\infty$.

8. Test the convergence of the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Solve $x^2y''+xy'-y=2x^2$, given that $\frac{1}{x}$ is a part of the complementary function.

2. Solve $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+(4\cos^2 x)y=0$ by changing the independent variable.

Contd.............2

3. Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}-(a^2+1)y=e^x\sec x$ by reducing to Normal form.

4. Solve $\frac{d^2y}{dx^2}+y=\sec x$ by the method of variation of parameters.

5. Test for the exactness and solve the equation $(1+x^2)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\sec^2 x$

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Solve $x^2\frac{d^2y}{dx^2}-(x^2+2x)\frac{dy}{dx}+(x+2)y=x^3e^x$ by finding complementary function.

2. Solve $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$ using the transformation $z=\sin^{-1}x$.

3. If $\lim_{n\to\infty}x_n=l$ and $\lim_{n\to\infty}y_n=m$, then prove that $\lim_{n\to\infty}(x_n-y_n)=l-m$

4. Discuss the nature of the sequence $\{n^{1/n}\}$.

5. Show that the sequence $\{S_n\}$, where $S_n=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+..........+\frac{1}{n(n+1)}$ is convergent.

IV. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Show that the sequence $\{S_n\}$ defined by $S_1=\sqrt{2}$ and $S_{n+1}=\sqrt{2S_n}$ converges to 2.

2. Find the limit of the sequence whose n^{th} term is $x_n=\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+............+\frac{n}{(n+1)!}$

3. Show that the sequence $\{S_n\}$ where n^{th} term is given by $S_n=\frac{2n-7}{3n+2}$ is (i) Monotonically increasing (ii) Bounded (iii) Tend to the limit $\frac{2}{3}$.

4. Discuss the convergence of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+..........$

5. Discuss the convergence of the series $\sum\frac{1}{n}\tan\frac{1}{n}$

Contd.............3

V. *Answer* any *THREE of the following:* 3 x 5 = 15 Marks

1. Test the convergence of the series $\sum \frac{2^{n-1}}{3^n+1}$

2. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2.5.8.........(3n-1)}{4.7.10.......(3n+1)}$

3. State and prove Cauchy's root test.

4. Test the convergence of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+......+.........$

5. Discuss the convergence of the series $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...........$ for all values of x.

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QP CODE 15532

FifthSemester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(Old Syllabus 2014 – 18)

MATHEMATICS

SSE 531: Paper–VI

Time: 3hrs.] [Max.Marks: 80

Note: i) Answer ALL questions.

ii) Use of non – programmable scientific calculator is allowed.

I. *Answer any TENof the following.* 10x2=20 Marks

1. Define Topological spaces (X, τ).

2. Find Four mutually non – comparable topologies for the set $X = \{a,b,c\}$

3. Define interior point.

4. Find the Fourier coefficient a_0 of the function $f(x) = x - 1$ in the interval $-\pi \le x \le \pi$.

5. Express $f(x) = \frac{1}{1-x}$ as a sum of even and odd function.

6. Define periodic function give an example.

7. Evaluate $\Delta^4\left[3x^4 + 2x^3 + 4x^2 + 7x + 9\right]$ taking $h = 1$.

8. Evaluate $\Delta e^{2x} \log 3x$.

9. Prove that $\Delta = E - 1$

10. If $u_0 = 3, u_1 = 12, u_2 = 81, u_3 = 200, u_4 = 100, u_5 = 8$ find the value of $\Delta^5 u_0$

11. Construct the dividend difference table for the given data

x	1	2	3	4	5
$f(x)$	0	10	38	96	196

12. Using Picard's method, find the first approximation to $\frac{dy}{dx} = x + y$ given $y = 1$ when $x = 0$

Contd......... 2

QP CODE 15532

II. ***Answer any THREE of the following:*** 3X5=15 Marks

1. Prove that the intersection of any number of topologies on 'X' is again a topology on 'X'.

2. Prove that if A and B be subsets of a topological space (X,τ) then $(A\cup B)' = A'\cup B'$

3. Let $X=\{a,b,c,d,e\}$, $\tau=\{X,\phi,\{a\},\{c,d\},\{a,c,d\},\{a,c,d,e\}$

   Find closure of (i) $A=\{b,c\}$

   (ii) $B=\{a,c\}$

   (iii) $C=\{a,b,c\}$

   (iv) $D=\{d\}$

4. Find $\text{int}(A), \text{ext}(A)$ and $b(A)$ of the subset $A=\{a,b,c\}$ of the set $X=\{a,b,c,d,e\}$ and $\tau=\{x,\phi,\{a\},\{b\},\{a,b\},\{a,c,d\},\{a,b,c,d\},\{a,b,c\}\}$

5. Let $X=\{a,b,c\}$ and , $\tau=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ be a topology for 'X'. If $B=\{\{a\},\{b\},\{c\}\}$ then show that B is a base of τ.

III. ***Answer any THREEof the following:*** 3x5=15 Marks

1. Obtain the Fourier series of the function $f(x)=\begin{cases}-\pi & -\pi<x<0\\ x & 0<x<\pi\end{cases}$ in the interval $(-\pi,\pi)$ andhence deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.........=\frac{\pi^2}{8}$

2. Obtain the Fourier series of the function $f(x)=\begin{cases}0 & -5<x<0\\ 3 & 0<x<5\end{cases}$ in the interval $(-5,5)$

3. Obtain the Fourier series of the function $f(x)=\frac{x^2}{4}$, $-\pi<x<\pi$

4. Find the half range cosine series for the function $f(x)=\sin x$ over the interval $(0,\pi)$

5. Find the half range sine series for the function $f(x)=x(\pi-x)$ in $(0,\pi)$

IV. *Answerany THREE of the following:* 3 x 5 = 15 Marks

1. Given $u_4 = 25, u_6 = 49, u_8 = 81$. Find the values of u_5 and u_7.

2. Find the real root of the equation $f(x) = x^3 - 4x - 9 = 0$ by Regula–Falsi method over the interval $(2, 5, 3)$ correct to 3 decimal places.

3. Find the value of 'θ' when $x = 38$ using the table

x	40	50	60	70	80	90
$f(x) = \theta$	184	204	226	250	276	304

Here the melting point of an alloy of lead and zinc is given, 'θ' is the temperature in degree centigrade and 'x' is the percentage of lead.

4. Estimate $f(9.7)$ from the table

x	8.0	8.5	9.0	9.5	10
$f(x)$	50	57	64	71	75

5. Given $f(1) = 2, f(2) = 4, f(3) = 8, f(7) = 128$ obtain the value of $f(5)$ using Lagrange's formula

V. *Answer* **any** *THREE of the following:* 3 x 5 = 15 Marks

1. For the following data, find $f'(x)$ and $f''(x)$ at $x = 53.5$

x	50	51	52	53	54
y	3.6840	3.7084	3.7325	3.7563	3.7798

2. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Simpson's $\frac{3}{8^{th}}$ rule by dividing the interval into three equal parts.

3. Evaluate $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$ using Weddle's rule using seven ordinates.

4. Using Euler's method, Solve $\frac{dy}{dx} = x^2 + y$ given $y = 0.94$ when $x = 0$ find 'y' for $x = 0(0.1)0.1$

5. Solve $\frac{dy}{dx} = 1 + y^2$ given $y(0) = 0$ find $y(0.2)$ by Runge – Kutta fourth order method.

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Contd........ 4

QP CODE 15532

Fifth Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(Before 2014 – 15 Old Syllabus)

MATHEMATICS

SSE 531: Paper – VI

Time: 3hrs.] [Max. Marks: 80

Note: i) Answer ALL questions.

ii) Use of non – programmable scientific calculator is allowed.

I. *Answer any TEN of the following.* 10 x 2 = 20 Marks

1. Find the part of the complementary function of $(x\sin x+\cos x)\,y''-x\cos y'+y\cos x=0$.

2. Verify the exactness of the equation, $x^2(1+x)\,y''+2x(2+3x)\,y'+2y(1+3x)=0$

3. Verify the condition for integribility of the equation $yzdx-2xzdy+(xy-3y^3)\,dz=0$

4. Solve $\frac{dx}{y^2}=\frac{dy}{x^2}=\frac{dz}{x^2yz^2}$

5. Form a partial differential equation by eliminating the constants a and b from the equation $z=ax+by+a^2+b^2$

6. Solve: $pq+p+q=0$

7. Show that $L[a^t]=\frac{1}{s-\log a}$

8. Find $L[\sin^2 t]$

9. Find $L^{-1}\left[\frac{s}{(s+4)^2}\right]$

10. State convolution theorem.

11. Find $L[6\cos 4t+3e^{-4t}]$

12. Express the function interms of unit step function $f(x)=\begin{cases}1 & 0<t<4\\ 2t+1 & t>4\end{cases}$

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Solve $x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x}, x>0$ given that e^x is a part of complimentary function.

2. Solve $\frac{d^2y}{dx^2}+(2\cos x+\tan x)\frac{dy}{dx}+(\cos^2 x)y=\cos^4 x$ by changing the independent variable.

3. Solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$ by changing the dependent variable.

4. Solve $\frac{d^2y}{dx^2}+4y=4\sec^2 2x$ by the method of variation parameter.

5. Verify the exactness of $xy''+3y'=\cos x$ and solve.

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Verify the condition for integrability and solve $3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$

2. Solve $\frac{dx}{y-z}=\frac{dy}{z-x}=\frac{dz}{x-y}$

3. Solve $zxp+yzq=xy$

4. Solve $p(1+q^2)=q(z-a)$

5. Solve $p^2-q^2=x-y$

IV. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. Find $L[\sin 6t.\cos 2t]$

2. Find a) $L\left[e^{at}(2t^2-3t+4)\right]$, b) $L\left[(a+bt)^3\right]$

3. Find the laplace transform of the function

$$f(t)=\begin{cases} E & 0\le t\le T/2 \\ -E & T/2\le t\le T \end{cases}$$

4. Find $L^{-1}\left[\frac{2s-1}{s^2-2s+10}\right]$

Contd......... 6

5. Solve the integral equation.

$$f(t) = at + \int_0^t f(u)\sin(t-u)\,du$$

V. ***Answer* any *THREE of the following:*** 3 x 5 = 15 Marks

1. Find $L^{-1}\left[\frac{s}{(s+1)^2(s^2+1)}\right]$ using convolution theorem.

2. Show that $L^{-1}\left[\frac{1}{(8^2+a^2)^2}\right] = \frac{1}{2a^3}[\sin at - at\cos at]$

3. Find $f(t)$, if $L[f(t)] = \log\left(\frac{s+a}{s-a}\right)$

4. Solve $y'' + 9y = 25e^{4t}$, given $y(0)=3,\ y'(0)=7$.

5. Solve $\frac{d^2y}{dt^2} + y = f(t)$ where $f(t) = \begin{cases} 0, & t<1 \\ 2, & t>1 \end{cases}$ $y(0)=0, y'(0)=0$.

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QP CODE 15531

FifthSemester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(Old Syllabus 2014 – 18)

MATHEMATICS

SSE 530: Paper–V

Time: 3hrs.] *[Max.Marks: 80*

Note: Answer ALL questions.

I. *Answer any TENof the following.* 10x2=20 Marks

1. Prove that the only idempotent elements of an integral domain are 0 & 1.
2. In a ring R if $a^2 = a \forall a \in R$, prove that $a + a = 0 \forall a \in R$.
3. Let $(R, +, \cdot)$ be a ring, then $\forall\ a, b \in R$

   i) $a(-b) = -(ab) = (-a)b$

   ii) $(-a)(-b) = ab$
4. Show by example that a ring 'R' without unity, may have a sub – ring 'S' with unity.
5. Prove that an ideal of a ring 'R' is a subring of the ring 'R'.
6. Let 'R' be the ring of all 2×2 matrices over the field of real numbers. Let 'M' be a subset of R and let elements of M matrices of type $\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$. Then prove that M is subring of 'R'.
7. Find $L\left[e^{3t}\, 10^t\right]$
8. Find $L\left[\sin^2 4t\right]$
9. Find $L[\cos 3t . \cos 4t]$
10. Find $f(t)$ if $L[f(t)] = \dfrac{s^4 + 3(s+1)^3}{s^4 (s+1)^3}$
11. Find $L^{-1}\left[\dfrac{5s+6}{(s+1)^2}\right]$
12. Find $L^{-1}\left[\cot^{-1}\left(\dfrac{s}{a}\right)\right]$

Contd.........2

QP CODE 15531

II. ***Answer any THREE of the following:*** 3X5=15 Marks

1. Prove that every finite integral domain is a field.
2. Prove that the ring of integer modulo – $n(Z_n, +_n, \times_n)$ is an integral domain iff 'n' is prime number.
3. Prove that the intersection of two subring is subring. Give an example to show "The union of two subrings is need not be a subring".
4. Let 'R' be commutative ring with unit element whose only ideals are $\{0\}$ and R itself. Then prove that 'R' is a field.
5. Let A and B are ideals of R, Let $A+B=\{u+v/u\in A, v\in B\}$. Then prove that $A+B$ is also an ideal of 'R'.

III. ***Answer any THREEof the following:*** 3x5=15 Marks

1. If 'P' is an integer then prove that PZ is maximal ideal of $(Z,+,\square)$ iff 'P' is prime.
2. Find all the principal ideals of the ring $(Z_{12}, +_{12}, X_{12})$
3. If I be the ideal of the ring R. Then the quotient ring R/I is homomorphic image of R with I as its kernel.
4. State and prove fundamental theorem of homomorphism.
5. Let $f: R \to R'$ is a homomorphism from ring R into R' with kernel K, then
   i) K is an subring of R
   ii) K is a ideal of R

IV. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. a) Find $L\left[(a+bt)^3\right]$

   b) Find $L[t\cos at]$ and $L[t\sin at]$
2. Prove that the Laplace transform of periodic function $f(t)$ with the period 'T' is,

   $$L[f(t)] = \frac{1}{1-e^{-ST}}\int_0^T e^{-st} f(t)\,dt$$
3. Find the Laplace transform of function,

   $$f(t)=\begin{cases} E, & for \quad 0\le t\le T/2 \\ -E, & for \quad T/2\le t\le T \end{cases}, \text{ for } f(t+T)=f(t)$$

Contd.........3

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Prove that a nonempty subset W of a vector space V is a subspace of V if and only if $c_1\alpha + c_2\beta \in W \ \forall \ \alpha, \beta \in W$ and $c_1, c_2 \in F$.

2. Prove that in an ordered set $s = \{\alpha_1, \alpha_2, \ldots\ldots\ldots \alpha_n\}$ vectors of a vector space $V(F)$ with $\alpha_1 \neq 0$ is linearly dependent iff one of the vector $\alpha_k \ (K > 1)$ is the linear combination of its preceding vectors.

3. In an n – dimensional vector space $V(F)$, show that
   i) Any set of $(n+1)$ vectors are linearly dependent
   ii) No set of $(n-1)$ vectors spans $V_n(F)$

4. In $V_3(Z_3)$ how many vectors are spanned by $(1,2,1)$ and $(2,1,1)$, find them?

5. Find the dimension and basis of the subspace spanned by the vectors $(1,2,0)\ (1,1,1)\ (2,0,1)$ of the vectors space $V_3(R)$, where Z_3 is the field of integers mod 3.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Examine whether transformation $T: R_3 \to R_3$ defined by $T(x_1, x_2, x_3) = (0, x_2, x_3)$ is linear or not.

2. Find the linear transformation $T: R^2 \to R^3$ where $T(1,1) = (0,1,2)$ and $T(-1,1) = (2,1,0)$.

3. Find the matrix of linear transformation $T: V_2(R) \to V_3(R)$, where $T(x,y) = (x+y, x, 3x-y)$ relative to the basis $B_1 = \{(1,1), (3,1)\}$ and $B_2 = \{(1,1,-1)\ (1,1,0)\ (1,0,0)\}$

4. If $T: V_3(R) \to V_3(R)$ be a linear transformation defined by $T(x,y,z) = (x+y, x-y, 2x+z)$ find range, null space, rank and nullity. Also verify Rank – Nullity theorem.

5. Find normal orthogonal basis of the subspace spanned by the vectors $(1,0,1)(1,0,-1)(0,3,4)$ in $V_3(R)$.

Contd.........6

IV. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. Show that $\int_C (x+y).dx+(x-y).dy=0$ where C is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

2. Evaluate $\int_C (3x^2-3yz+2xz).dx+(3y^2-3xz+z^2)dy+(3z^2-3xy+x^2+2yz).dz$ where C is the curve leading from $(-1, 2, 3)$ to $(3, 2, -1)$.

3. Evaluate $\iint_R y.dx.dy$ where R is the region bounded by the parabolas $y^2=4x$ and $x^2=4y$

4. Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}dx.dy$ where R is a annular region between two circles $x^2+y^2=2$ and $x^2+y^2=1$.

5. Find the area of the surface of the position $z=x^2+y^2$ between $z=2$ and $z=6$.

V. ***Answer* any *THREE of the following:*** 3 x 5 = 15 Marks

1. Find volume of the surface $z=a^2-x^2$ and the plane $x=0, y=0, z=0$ and $y=b$

2. Evaluate $\iiint xyz.dx.dy.dz$ over the positive octant of a sphere by converting it in to cylindrical co-ordinates.

3. Prove that $\beta(m, n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

4. Solve $\int_0^a x^4\left(\sqrt{a^2-x^2}\right).dx$

5. Show that $\int_0^1 x^m(1-x^n)^p.dx=\frac{1}{n}\beta\left(\frac{m+1}{n}, p+1\right)$ and hence evaluate $\int_0^1 x^5(1-x^3)^{10}.dx=\frac{1}{396}$

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Fifth Semester B.Sc. Degree Examinations

APRIL /MAY 2022

(Semester Scheme)

(Before 2014 – 15 Old Syllabus)

MATHEMATICS

SSE 530: Paper – V

Time: 3hrs.] [Max. Marks: 80

Note: Answer ALL questions.

I. *Answer any TEN of the following.* 10 x 2 = 20 Marks

1. In any vector space $V(f)$, show that $(c_1+c_2)\vec{\alpha}=c_1\vec{\alpha}+c_2\vec{a}$, where $c_1,c_2\in F$ and $\vec{\alpha}\in v$
2. Express the vectors $(3,5,2)$ as a linear combination of vectors $(1,1,0)(2,3,0)(0,0,1)$ of $V_3(R)$.
3. If $W=\{(x,0,0)/x\in R\}$ be the subset of $V_3(R)$, show that W is a subspace of $V_3(R)$.
4. If $T:V_1(R)\to V_3(R)$, defined by $T(x)=(x,x^2,x^3)$. verify whether T is linear or not.
5. If $T:V\to V$ be a linear transformation, then prove that $T(-\alpha)=-T(\alpha)$
6. Prove that $|\alpha+\beta|\le|\alpha|+|\beta|$ in an euclidean vectors space.
7. Evaluate $\int_C y.dx-x.dy-z^2.dz$, where C is the curve, and $x=\sin t, y=\cos t, z=t$ and $0\le t\le 1$
8. Evaluate $\int_0^1\int_0^{(1-x)} xy.\,dy.dx$
9. Evaluate $\int_0^1\int_0^1\int_0^y xyz.\,dx.dy.dz$
10. Prove that $\Gamma(n+1)=n.\Gamma(n)$
11. Simplify $\dfrac{6\Gamma(8/3)}{5\Gamma(2/3)}$
12. Show that $\int_0^{\pi/2}\sqrt{\tan\theta}.d\theta=\dfrac{\pi}{\sqrt{2}}$

4. a) If $L[f(t)] = F(s)$ then prove that $L[t.f(t)] = -F'(s)$

   b) Find $L[t\sin^3 3t]$

5. a) If $L[f(t)] = F(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$

   b) If $L[f(t)] = F(s)$, then prove that $L[f(kt)] = \frac{1}{k}\, F\left(\frac{s}{k}\right)$.

V. ***Answer* any *THREE of the following:*** 3 x 5 = 15 Marks

1. Show that the inverse of Laplace transform of $\frac{s^2}{s^4+4a^4}$ is

   $\frac{1}{2a}[\sinh at\cos at + \cosh at\sin at]$

2. State and prove convolution theorem.

3. Express $f(t) = \begin{cases} t^2 & 1 < t \le 2 \\ 4t & t > 2 \end{cases}$ in terms of unit step function and hence find $L[f(t)]$

4. Solve $f(t) = \sin t + 5\int_0^t f(u).\sin(t-u)\,du$

5. Solve $9y'' - 6y' + y = 0$, given that $y(0) = 3$ and $y'(0) = 1$.

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Contd.........4

Fifth Semester B.Sc. Degree Examinations

SEPTEMBER/OCTOBER 2022

(Semester Scheme) (New Syllabus 2018 – 19 Onwards)

MATHEMATICS

SSE 541: Paper – BSM 6: LINE AND MULTIPLE INTEGRALS AND LAPLACE TRANSFORMS

Time: 3 hrs.] *[Max. Marks: 70*

Note: Answer ALL questions.

I. ***Answer any FIVE of the following.*** 5 x 2 = 10 Marks

1. Evaluate $\int_C (x+y)\,dx+(y-x)\,dy$ along the parabola $y^2=x$ from $(1,1)$ to $(4,2)$

2. Evaluate $\int_0^1\int_0^{1-x} xy\,dy\,dx$

3. Evaluate $\iint_D x^2\,dx\,dy$ where D is the region bounded by the trapezium where sides are the lines $y=0, y=1, x=0$ and $x=5-2y$.

4. Evaluate $\int_0^a\int_0^a\int_0^a (x^2+y^2+z^2)\,dx\,dy\,dz$

5. Find $L[\sin 3t\ \cos 4t]$

6. Find Laplace transform of t^2c^{2t}

7. Evaluate $L^{-1}\left[\dfrac{s}{(s+9)^2}\right]$

8. Show that $L[u(t-a)]=\dfrac{e^{-as}}{s}$, where $u(t-a)$ is unit step function.

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Evaluate $\int_C [xydx+x^2zdy+xyzdz]$ where C is the curve $x=e^t$, $y=e^{-t}, z=t^2$ and $0\le t\le 1$.

2. If C is the curve leading from $(-1,2,3)$ to $(3,2,-1)$. Then evaluate $\int_C (3x^2-3yz+2xz)\,dx+(3y^2-3xz+z^2)\,dy+(3z^2-3xy+x^2+2yz)\,dz$.

3. Evaluate $\iint_D xy(x+y)\,dx\,dy$ over the domain D between the parabola $y=x^2$ and the line $y=x$.

4. Evaluate $\int_0^2\int_0^{\sqrt{4-y^2}} \sqrt{\left(4-x^2-y^2\right)}\,dx\,dy$

5. Evaluate $\iint_D \frac{x^2y^2}{\left(x^2+y^2\right)^2}\,dx\,dy$, where D is the region $1<x^2+y^2<2,\ 0<\theta<\pi/2$

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \left(x^2+y^2\right)dy\,dx$ by changing into polar coordinates.

2. Find the area of the surface $z=\sqrt{x^2+y^2}$, where $\frac{1}{9}<x^2+y^2<\frac{1}{4}$.

3. Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

4. Find the volume bounded by the surface $z=16-x^2$ and the planes $x=0, y=0, z=0$ $y=3$.

5. Find the volume common to the cylinders $x^2+y^2=R^2$ and $x^2+z^2=R^2$.

IV. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. a) Evaluate $L\left[e^{-3t}\left(2\cos 5t-3\sin 5t\right)\right]$

   b) Find the Laplace transform of $L\left[t^2\sin t\right]$

2. If $L\left[f(t)\right]=F(s)$ then prove that $L\frac{\left[f(t)\right]}{t}=\int_s^\infty F(s)\,ds$

3. Find the Laplace transform of $f(t)=\frac{Kt}{P}$ for $0<t<P$ and $f(t+p)=f(t)$.

4. Express $f(t)$ in terms of unit step function and also find $L\left[f(t)\right]$, where $f(t)=\begin{cases}1, & 0<t<4\\ 2t+1, & t>4\end{cases}$

5. Evaluate $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$

V. ***Answer* any *THREE of the following:*** 3 x 5 = 15 Marks

1. Find $L^{-1}\left[\frac{2s+3}{(s-1)(s+2)^2}\right]$

2. Find $f(t)$. Given that $F(s)=\log\left[\frac{s^2+1}{s(s+1)}\right]$

3. Using convolution theorem, find the increase Laplace transform of $\frac{1}{(s^2+a^2)^2}$

4. Solve the integral equation

$$f(t)=Q\sin t-2\int_0^t f(u)\cos(t-u)\,du.$$

5. Solve using Laplace transformation $9y''-6y'+y=0$ given $y(0)=3$ and $y'(0)=1$.

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Fifth Semester B.Sc. Degree Examinations

SEPTEMBER/OCTOBER 2022

(Semester Scheme) (New Syllabus 2018 – 19 Onwards)

MATHEMATICS

SSE 540: Paper – BSM V: DIFFERENTIAL EQUATIONS – III, FOURIER SERIES AND ALGEBRA – IV.

Time: 3 hrs.] [Max. Marks: 70

Note: Answer ALL questions.

I. ***Answer any FIVE of the following.*** 5 x 2 = 10 Marks

1. Verify the condition of integrability for the equation $z^2\,dx+\left(z^2-2yz\right)dy+\left(2y^2-yz-xz\right)dz=0$.

2. Solve $\frac{dx}{y^2}=\frac{dy}{x^2}=\frac{dz}{x^2yz^2}$

3. Form the partial differential equation by eliminating arbitrary constants a & b from $z=axe^y+\frac{1}{2}a^2e^{2y}+b$.

4. Solve $z=pq$

5. Find the Fourier coefficient a_0 for the function $f(x)=x-1$ in the interval $(-\pi,\pi)$

6. Check whether the function. $f(x)$ defined by $f(x)=\begin{cases} x+1 & in \quad (-\pi,0) \\ x-1 & in \quad (0,\pi) \end{cases}$ is even or odd.

7. Define a division ring and give an example of a division ring which is not a field.

8. Let R be a commutative ring and I be an ideal of R. Then show that R/I is also commutative.

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Verify the condition of integrability and solve $yz\log z\,dx-zx\log z\,dy+xy\,dz=0$

2. Verify the condition of integrability and solve $3x^2dx+3y^2dy-\left(x^3+y^3+e^{2z}\right)dz=0$

Contd............2

3. Solve $\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$

4. Solve $\frac{dx}{z^2-2yz-y^2}=\frac{dy}{xy+zx}=\frac{dz}{xy-zx}$

5. Solve $(x+2z)p+(2zx-y)q=x^2+y$

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Solve $z(p^2-q^2)=x-y$ using the substitution $\frac{2}{3}z^{3/2}=Z$.

2. Find the complete and singular solution of $z=px+qy+c\sqrt{1+p^2+q^2}$

3. Solve $pxy+pq+qy=yz$ by Charpit's method.

4. Find the Fourier series of the function $f(x)=x-x^2$, in $(-1,1)$.

5. Obtain Fourier series of the following function $f(x)=\begin{cases} x-\frac{\pi}{2} & in \quad -\pi<x<0 \\ x+\frac{\pi}{2} & in \quad 0<x<\pi \end{cases}$

IV. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Find the half range cosine series of $f(x)=\begin{cases} x & if \quad 0<x<\frac{\pi}{2} \\ \pi-x & if \quad \frac{\pi}{2}<x<\pi \end{cases}$ in $(0,\pi)$

2. Determine the half range sine series of the function $f(x)=(\pi-x)^2$ in $(0,\pi)$

3. Show that a finite integral domain is a field.

4. Show that the union of two sub rings of a ring R is a sub ring of R if and only if one is contained in the other.

5. Show that the set of matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ where $x\in R$ is a sub ring of $M_2(R)$. Show that unit element of ring and sub ring are different. Also show that every non zero element of sub ring has multiplicative inverse.

Contd...............3

V. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Show that a commutative ring with unity is a field if and only if it has no proper ideals.

2. Find all principal and maximal ideals of $(z_{12}, +_{12}, \times_{12})$.

3. Prove that the only homomorphism from a ring of integers Z to Z is either the identity map or a zero map.

4. If f is a homomorphisms form a ring R into a ring R' with K as its kernel then show that $f(R)$ is isomorphic to the quotient ring R/K.

5. Let R be a commutative ring with unit element and S be an ideal of R such that R/S is a field. Then show that S is a maximal ideal of R.

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Sixth Semester B.Sc. Degree Examinations

SEPTEMBER/OCTOBER 2022

(Semester Scheme) (New Syllabus 2018 Onwards)

MATHEMATICS

SSF 541: Paper – VIII: BSM 8: REIMANN INTEGRATION, VECTOR CALCULUS AND COMPLEX ANALYSIS

Time: 3 hrs.] *[Max. Marks: 70*

Note: Answer ALL questions.

I. ***Answer any FIVE of the following.*** 5 x 2 = 10 Marks

1. Prove that the constant function $f(x)=K$, where K is constant is a R – integrable over $[a,b]$
2. Prove that the Lower Reimann integral cannot exceed the upper Reimann integral.
3. If $\phi(x,y,z)=xy^2z^3-x^3y^2z$ then find $\nabla\phi$ at the point $(1,-1,1)$
4. Prove that $\nabla.(f+g)=\nabla.f+\nabla.g$.
5. Show that $|z-1|^2+|z+1|^2=4$ represent a unit circle.
6. Evaluate: $\lim\limits_{z\to 1+i}\left(z^2-5z+10\right)$
7. Prove that $f(z)=\sin z$ is analytic function
8. Show that $u=e^x\sin y+x^2-y^2$ is harmonic function.

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Prove that $L(P,f)$ and $U(P,f)$ are bounded.
2. Prove that every monotonic function is $R-$integrable.
3. Prove that the function $f(x)=3x+1$ is $R-$integrable over the interval $[1,2]$
4. If $f(x)$ is bounded and $R-$integrable over $[a,b]$, then prove that $|f(x)|$ is also $R-$integrable over $[a,b]$ and $\left|\int_a^b f(x)dx\right|\le\int_a^b|f(x)|\,dx$.

Contd...............2

5. If $f(x)=\frac{1}{3^n}$ for $\frac{1}{3^{n+1}}<x\le\frac{1}{3^n}$ for $n=0,1,2,3,\ldots\ldots$ and $f(0)=0$, then show that $f(x)$ is $R-$integrable over $[0,1]$ and hence show that $\int_0^1 f(x)\,dx=\frac{3}{4}$.

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the points $(2,-1,2)$ common to them.
2. Find the directional derivative of $\phi(x,y,z)=xy^2+yz^3$ at $(2,-1,1)$ in the direction $2i+j+2k$.
3. If $\vec{r}=xi+yj+zk$ and $r=|\vec{r}|$, then show that $div(r^n.\vec{r})=(n+3)r^n$.
4. Show that $\vec{f}=(2xy+z^3)i+x^2j+3xz^2k$ is irrotational and find the function ϕ such that $\vec{f}=grad\,\phi$.
5. Prove that $curl(grad\,\phi)=0$

IV. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Show that $Arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represent a circle. Find its centre and radius.
2. Find the derivative of $w=f(z)=z^3-2z$ at the point $z=z_0$, by using the definition of the derivative.
3. Find the equation of the circle passing through the points $1,i,1+i$ and also find its centre and radius.
4. Evaluate $\lim\limits_{z\to 1+i}\frac{z^2-z+1-i}{z^2-2z+2}$
5. Verify the function $f(z)=\frac{z^2+4}{z-2i}$ if $z\neq 2i$ and $f(2i)=3+4i$ is continuous at $z=2i$.

V. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. State and prove sufficient condition for the function $f(z)$ to be analytic.
2. Show that $f(z)=\log z$ is analytic and hence prove that $f'(z)=\frac{1}{z}$.

Contd…………3

3. If $f(z)=u(x,y)+iv(x,y)$ is analytic function in the complex plane, then prove that $u(x,y)=c_1$, $v(x,y)=c_2$ where c_1 and c_2 are constants, represents the orthogonal family of curves.

4. If $f(z)=u+iv$ is analytic and u,v are harmonic functions, then show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

5. Construct the analytic function whose real past $u=e^x \sin y$. Also find its imaginary part.

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QP CODE 15631

Sixth Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(Before 2014 – 15 Syllabus)

MATHEMATICS

SSF 530: Paper – VII

Time: 3hrs.] [Max. Marks: 80

Note: i) Answer ALL questions.

ii) Use of non – programmable scientific calculator is allowed.

I. *Answer any TEN of the following.* 10 x 2 = 20 Marks

1. Show that the intersection of two convex sets is a convex set.
2. Draw the graph of the region $5x_1 + 10x_2 \leq 50,\ x_1 + x_2 \geq 1,\ x_2 \leq 4$ and $x_1, x_2 \geq 0$
3. Define slack variable and surplus variable.
4. Express $f(x) = e^x$ as a sum of even and odd function.
5. Find a_0 for $f(x) = x - x^2$ where $-\pi < x < \pi$
6. Find the half range sine series of $f(x) = 1$ in $0 < x < 1$
7. Prove that $\nabla = \Delta E^{-1}$ and $E = 1 + \Delta$
8. Evaluate $\left(\dfrac{\Delta^2}{E}\right)e^x \cdot \dfrac{Ee^x}{\Delta^2 e^x}$
9. Construct the forward difference table for $f(x) = x - 2x^3$ for $x = 0(1)4$.
10. Solve $\dfrac{dy}{dx} = x + y$ given $y = 1$ when $x = 0$ using Picard's method upto second approximation.
11. Evaluate $\int_0^1 y\,dx$ using Trapezoidal rule given that

x	0	0.2	0.4	0.6	0.8	1.0
y	1	1.2214	1.4918	1.8221	2.2255	2.7183

Contd............6

12. Use Euler's method to solve $\frac{dy}{dx} = 1 - y, y(0) = 0, h = 0.1$, find $y(0.1)$

II. ***Answer any THREE of the following:*** 3 X 5 = 15 Marks

1. Find all the feasible solution of the system $x + 2y + z = 4, 2x + y + 5z = 5$. Are the solution degenerate?

2. Solve graphically the following LPP
   Minimize: $z = 7x + 8y$
   Subject to the constraints : $3x + y \geq 8,\ x + 3y \leq 11$
   and $x, y \geq 0$

3. The production of two types of watches, a factory uses three machines A, B and C. The time required for each watch on each machine and maximum time available on each machine is given below.

Machines	Time required for		Maximum time available
	Watch I	Watch II	
A	6	8	380
B	8	4	300
C	12	4	404

The profit on Watch I is ₹ 50 and on Watch II is ₹ 30 what combination should be produced for the maximum profit? What is the maximum profit?

4. Maximize: $z = 4x + 5y$
   Subject to constraints: $x + 7y \leq 28,\ x + y \leq 10$, and $x, y \geq 0$
   by simplex method.

5. Minimize: $z = 2x + 5y$
   Subject to constraints: $3x + 3y \geq 5,\ x + 4y \geq 6$ and $x, y \geq 0$
   by simplex method.

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Obtain the Fourier series of $f(x) = e^{-ax}, -\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

2. Find the Fourier series of $f(x) = |x|$ in the interval $-\pi < x < \pi$. Hence deduce
   $$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

3. Find the Fourier series of $f(x) = 2x - x^2$ on the interval $[0, 2]$.

Contd.........7

IV. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Compute all the basic feasible solutions to the LPP,

Maximize; $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$

Subject to constraints $2x_1 + 3x_2 - x_3 + 4x_4 = 8$

$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$

Where $x_1, x_2, x_3, x_4 \geq 0$ and check that one maximizes 'z'.

2. Solve graphically the following LPP

Maximize; $Z = 3x + 5y$

Subject to constraints : $3x + 2y \leq 18$

$x \leq 4$

$y \leq 6$ and $x, y \geq 0$

3. In the product of two types of watches in a factory uses three machines A, B, C. The time required for each watch or each machines and the maximum time available of each machine is given below.

Machine	Time required for		Maximum time available
	Watch I	Watch II	
A	6	8	380
B	8	4	300
C	12	4	404

The profit on watch I is ₹ 50 and on watch II is ₹ 30. What combination should be produced for the maximum profit and what is the maximum profit.

4. Solve the LPP

Maximize; $Z = 5x + 3y$

Subject to constraints $x + y \leq 2$

$5x + 2y \leq 10,$

$3x + 8y \leq 12,$

and $x, y \geq 0$

by Simplex method.

5. Solve the LPP by using Simplex method.

Maximize; $Z = 2x + 3y$

Subject to constraints $x + y \geq 5$

$x + 2y \geq 6$ and $x, y \geq 0$

Contd.........4

V. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Evaluate $\int_2^3 (3x+2)\,dx$ by using integration as a limit of sum.

2. State and prove Darboux's theorem.

3. Prove that every monotonic function is R – integrable on $[a,b]$

4. Show that the function $f(x)=3x+1$ is R – integrable over $[2,3]$ and $\int_2^3 (3x+1)\,dx=\frac{17}{2}$

5. Show that the function $f(x)=\frac{1}{2^n}$ where $\frac{1}{2^{n+1}}<x<\frac{1}{2^n}, \forall n=0,1,2........$ and $f(0)=0$ is R – integrable over $[0,1]$ and $\int_0^1 f(x)\,dx=\frac{2}{3}$.

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Contd.........5

SixthSemester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(2014 – 18OldSyllabus)

MATHEMATICS

SSF 530: Paper–VII

Time: 3hrs.] [Max.Marks: 80

Note: Answer ALL questions.

I. *Answer any TENof the following.* 10x2=20 Marks

1. In a vector space V, prove that $C.\alpha = 0 \Rightarrow C = 0$ or $\alpha = 0$, where $C \in F, \alpha \in V$.
2. Determine whether $W = \{(x, y, z)/x + y + z = 0\}$ is a subspace of $V_3(R)$ or not.
3. Show that the set $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is linearly independent in $V_3(R)$
4. Determine whether the transformation $T : V_1(R) \to V_3(R)$ defined by $T(x) = (x, x^2, x^3)$ is linear or not.
5. Find matrix of linear transformation $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (x + y, x, 3x - y)$ with standard basis.
6. In an Euclidean vector space, prove that $|\alpha| - |\beta| \le |\alpha - \beta|$
7. Define De – generate solution and optimal solution.
8. Represent $x_1 > 4$ and $x_2 > -2$ system graphically and find its solution set.
9. Define convex set and give an example.
10. Prove that $L(p, f) \le U(p, f)$
11. Compute $U(p, f)$ and $L(p, f)$ for the function $f(x) = x^2 + 1$ defined on $[0,1]$ with respect to the partition $P = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$.
12. If $f(x)$ is continuous in $[a,b]$ there is a point ξ, $a < \xi < b$ then prove that $\int_a^b f(x)\,dx = (b - a) f(\xi)$

II. *Answer any THREE of the following:* 3X5=15 Marks

1. Prove that the intersection ofany two subspaces of vector space 'V' over 'F' is also a subspace of 'V' over 'F'. Give an example to show "The union of two subspaces of a vector space V is need not be a subspace of 'V'.

2. Express the vector $(1,-2,5)$ as a linear combination of the vectors $(1,1,1),(1,2,3)$ and $(2,-1,1)$

3. In $V_3(Z_3)$ how many vectors are spanned by the vectors $(1,2,1)$ and $(2,1,2)$ and find them.

4. If 'n' non – zero vectors of a vector space V spans V and 'r' vectors of V are linearly Independent, then prove that $n \geq r$.

5. Find the basis and dimension of the subspace spanned by $S=\{(1,1,1),(2,1,2),(1,0,1),(5,3,5)\}$ in $V_3(R)$

III. *Answer any THREEof the following:* 3x5=15 Marks

1. Find the linear transformation $T: V_2(R) \to V_2(R)$ such that $T(1,1)=(0,1)$ and $T(-1,1)=(3,2)$.

2. Prove that if $\beta_1, \beta_2, \beta_3 \ldots \ldots \beta_m$ be any basis of vector space 'V' and $\alpha_1, \alpha_2, \alpha_3 \ldots \ldots \alpha_m$ be any 'm' vectors of the vectorspace 'W' then there exists one and only one linear transformation $T: V \to W$ with $T(\beta_i)=\alpha_i$, where $i=1,2,3 \ldots \ldots m$.

3. Find the linear transformation for the matrix $A=\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$ relative to the basis $B_1=\{(0,1),(-1,1)\}$ and $B_2=\{(1,1,1),(1,-1,1),(0,0,1)\}$

4. Verify the Rank – Nullity theorem for the transformation $T: V_3(R) \to V_2(R)$ defined by $T(x,y,z)=(y-x, y-z)$

5. Find the normal orthogonal basis for the subspace spanned by $(2,0,0,0),(1,3,3,0)$ and $(0,4,6,1)$.

Contd........ 3

4. Find the cosine half range series of $f(x) = x\sin x$ in $0 < x < \pi$

5. Find the half range sine series of $f(x) = \begin{cases} x & 0 \le x \le \pi/2 \\ \pi - x & \pi/2 \le x \le \pi \end{cases}$ and hence deduce

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots$$

IV. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. If $u_0 = 25$, $u_2 = 49$ & $u_4 = 81$ and 3rd order difference is zero, find values of u_1 and u_3.

2. Find the cubic polynomial which takes the following values.

x	0	1	2	3
$f(x)$	1	2	1	10

3. From the following data. Calculate the increase of population during the period 1955 to 1961.

Year	1921	1931	1941	1951	1961	1971
Population in Lakhs	20	24	29	36	46	51

4. Find the weight of the baby at seven months from the following data.

Age in months	0	2	5	8
Weight in pounds	6	10	12	16

by using Lagrange'ş method.

5. Find $f'(1.5)$ and $f''(1.5)$ from the following data.

x	1.5	2	2.5	3	3.5	4
$f(x)$	3.375	7	13.625	24	38.875	59

V. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Evaluate $\int_0^1 \frac{4^x}{x+1} dx$, by using Simpson's $1/3^{rd}$ rule by dividing the interval into six equal parts.

2. Evaluate $\int_1^4 e^{1/x}\,dx$, by using Simpson's $3/8^{th}$ rule by dividing the interval into six equal parts.

3. Using Picard's method of successive approximations, find the solution of $\frac{dy}{dx} = x - y,\ y(0) = 1$ for $x = 0.2$ upto fourth approximation.

4. Using Modified Euler's method solve $\frac{dy}{dx} = x + y$, where $y = 1$ when $x = 0$ for $x = 0.1$

5. Solve $\frac{dy}{dx} = 1 + y^2,\ y(0) = 0$ for $x = 0.2$ using Runge Kutta method.

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Sixth Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme) (New Syllabus 2018 Onwards)

MATHEMATICS

SSF 540: Paper – BSM 7: VECTOR SPACE AND NUMERICAL ANALYSIS

Time: 3 hrs.] [Max. Marks: 70

Note: i) Answer ALL questions.
ii) Scientific Calculator is allowed.

I. *Answer any FIVE of the following.* 5 x 2 = 10 Marks

1. In any vector space $V(F)$, show that $C\alpha = C\beta$ and $C \neq O \Rightarrow \alpha = \beta, \forall \alpha, \beta \in V$ and $C \in F$.

2. Is the subset $W = \{(x, y, z) / x^2 + y^2 + z^2 \leq 0\}$ a subspace of $V_3(R)$? Justify.

3. Verify whether set $\{(1,2,1),(3,4,-7),(3,4,5)\}$ is a basis of $V_3(R)$ or not?

4. Show that the mapping $T: V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (x, y)$ is a linear transformation.

5. Show that $\nabla = \Delta E^{-1}$.

6. Evaluate $\Delta^3[(1+3x)(1-2x)(1+4x)$ for $h = 1$

7. Using Picard's method, find the second approximation to $\frac{dy}{dx} = 1 + xy$ given $y = 0$ when $x = 0$.

8. Using Simpson's $1/3^{rd}$ rule, Evaluate $\int_0^3 \frac{dx}{1+x^2}$ given that

x	0	1	2	3
$f(x)$	1	0.5	0.2	0.1

Contd.............2

QP CODE 15673

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Prove that intersection of two subspaces of a vector space $V(F)$ is again a subspace of $V(F)$. Give an example to show that the union need not be a subspace.

2. Let $S=\{(1,-3,2),(2,4,1),(1,1,1)\}$ be a subset of $V_3(R)$. Show that the vector $(3,-7,6)$ is in $L[S]$.

3. In an n – dimensional vector space $V(F)$, then prove that (i) any $(n+1)$ elements of V are linearly dependent. (ii) No set of $(n-1)$ elements can span V.

4. In $V_3(R)$, show that the plane $x_3=0$ may be spanned by the pair of vectors $(2,2,0)$ and $(4,1,0)$.

5. Find the dimension and basis of the subspace spanned by the vectors $(2,4,2),(1,-1,0),(1,2,1)$, and $(0,3,1)$ in $V_3(R)$.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Verify whether the transformation $T:R^2 \to R^3$ defined by $T(x,y)=(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$

2. Find the linear transformation $f:V_2(R)\to V_3(R)$ such that $T(1,1)=(0,1,2)$ and $T(-1,1)=(2,1,0)$.

3. Find the matrix of linear transformation $T:V_3(R)\to V_2(R)$ relative to the bases $B_1=\{(1,1,1),(1,0,0),(1,1,0)\}$ and $B_2=\{(1,0),(0,1)\}$

4. State and prove Rank – Nullity theorem.

5. Find the range, nullspace, rank and nullity of linear transformation $T:V_3(R)\to V_3(R)$ defined by $T(x,y,z)=(x+y,x-y,2x+z)$.

IV. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Find a real positive root of the equation $x^3-7x+5=0$ using bisection method in five stages.

2. Find the cube root of 24, correct to three places of decimal by Newton – Raphson method.

Contd...........3

3. Given

x	1	2	3	4	5	6
y	1	8	27	64	125	216

Estimate $f(2.5)$

4. The following table gives the normal weights of babies during the first few months of life.

Age in Months	2	5	8	10	12
Weight in Kgs	4.4	6.2	6.7	7.5	8.7

Estimate by Lagrange's method the normal weight of a baby of 7 months old.

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 54$ from the following table.

x	50	51	52	53	54
y	3.6840	3.7084	3.7325	3.7563	3.7798

V. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Using Simpson's $3/8$ rule, obtain an approximate value of $\int_0^{0.3}(2x-x^2)^{1/2}\,dx$.

2. Evaluate $\int_4^{5.2}\log x$ by using Weddle's rule by dividing the internal $(4,5.2)$ into six equal parts.

3. Using Picard's method of successive approximation, solve $\frac{dy}{dx} = x^2 + y^2$ given that $y(0)=0$ for $x=0.4$ upto third approximation.

4. Solve $\frac{dy}{dx} = x + y$ given that $x_0 = 0,\ y_0 = 1$ for $x = 0.05$ using Euler's modified method correct upto three decimal places.

5. Find the approximate solution at $x = 1.2$ of the equation $\frac{dy}{dx} = xy$ given $y(1) = 2$ by Runge – Kutta method.

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Sixth Semester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme) (New Syllabus 2018 Onwards)

MATHEMATICS

SSF 541: Paper – BSM VIII: REIMANN INTEGRATION, VECTOR CALCULUS AND COMPLEX ANALYSIS

Time: 3 hrs.] [Max. Marks: 70

Note: Answer ALL questions.

I. *Answer any FIVE of the following.* 5 x 2 = 10 Marks

1. If $P=\left[0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right]$ be a partition of $[0,1]$. Find $L(P,f)$ and $U(P,f)$ for the function $f(x)=2x$.
2. State Darboux theorem.
3. If $\phi(x,y,z)$ is a scalar point function, then prove that $div(grad\phi)=\nabla^2\phi$
4. If $\vec{f}=(ax+3y+4z)i+(x-2y+3z)j+(3x+2y-z)k$ is solenoidal vector field. Then find the value of a.
5. Find the locus of the point satisfying the relation imaginary $(z+i)\geq 0$
6. Evaluate: $\lim\limits_{z\to e^{i\frac{\pi}{4}}}\dfrac{z^2}{z^4+z+1}$
7. If $f(z)$ is analytic function, such that $f(z)$ is always real then show that f is constant.
8. Show that the function $u(x,y)=x^3-3xy^2$ is harmonic.

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Let f be a real valued bounded function defined on $[a,b]$ and let m and M be infimum and supremum of $f(x)$ in $[a,b]$. Then prove that for any partition P of $[a,b], m(b-a)\leq L(P,f)\leq U(P,f)\leq M(b-a)$
2. Show that $f(x)=2x+3$ is $R-$integrable on $[0,1]$ and hence prove that $\int_0^1 f(x)dx=4$

Contd.............2

3. Prove that every monotonic function is $R-$integrable.

4. Evaluate $\int_{2}^{3}(3x+2)\,dx$ by using integration as limit sum.

5. Show that the function $f(x)=\frac{1}{3^n}$ where $\frac{1}{3^{n+1}}<x<\frac{1}{3^n}$ for all $n=0,1,2,......$ and $f(0)=0$ is $R-$integrable over $[0,1]$ and $\int_{0}^{1}f(x)\,dx=\frac{3}{4}$

III. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Show that the surfaces $4x^2-z^3=4$ and $5x^2-2yz=7x$ intersect orthogonally at the point $(1,-1,-2)$

2. Find the directional derivative of $\phi(x,y,z)=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction of $2i+j+2k$. Also find maximal directional derivative of ϕ at $(2,-1,1)$.

3. Show that $\vec{f}=(\sin y+z)i+(x\cos y-z)j+(x-y)k$ is irrotational and find the function ϕ such that $\vec{f}=grad\,\phi$.

4. If $\vec{r}=xi+yj+zk$ and $r=|\vec{r}|$, then show that $div\left(r^n\vec{r}\right)=(n+3)r^n$.

5. Let $\phi(x,y,z)$ be a scalar point function and $\vec{f}=f_1i+f_2j+f_3k$ be a vector point function, then prove that $curl\left(\phi\cdot\vec{f}\right)=\phi\cdot curl\,\vec{f}+(grad\,\phi)\,X\,\vec{f}$

IV. ***Answer any THREE of the following:*** 3 x 5 = 15 Marks

1. Prove that $|z_1\cdot z_2|=|z_1||z_2|$ and $amp(z_1\cdot z_2)=amp z_1+amp z_2$

2. Show that $Arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represent a circle, Find its centre and radius.

3. Find the equation of the circle passing through the points $1,i,1+i$. Find its centre and radius.

4. Derive the equation of a straight line passing through the two points z_1 and z_2 in complex form.

5. Using the definition find the derivative of $f(z)=z^3$ at $z=z_0$

Contd..............3

V. *Answer* any *THREE of the following:* 3 x 5 = 15 Marks

1. Prove that the necessary condition for $f(z)=u(x,y)+iv(x,y)$ to be analytic is that, $f(z)$ satisfies the C – R equations.
2. Show that $f(z)=\cos z$ is analytic and hence show that $f'(z)=-\sin z$
3. Show that the function $u=x^3-3xy^2$ is harmonic. Find its harmonic conjugate.
4. Construct an analytic function whose real part is $u=e^x \sin y$ Also find its imaginary part.
5. If $f(z)=u+iv$ is analytic, then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}|f(z)|\right]^2=|f'(z)|^2$

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SixthSemester B.Sc. Degree Examinations

APRIL / MAY 2022

(Semester Scheme)

(Old Syllabus 2014 – 18)

MATHEMATICS

SSF 531: Paper–VIII

Time: 3hrs.] [Max.Marks: 80

Note: Answer ALL questions.

I. *Answer any TENof the following.* 10x2=20 Marks

1. Evaluate $\lim_{z\to(1-i)} \left[z^2+2z\right]$

2. Find $\lim_{z\to 1+i} \left[\dfrac{z-1-i}{z^2-2z+2}\right]$

3. Show that $\left|\dfrac{z-1}{z+1}\right| = 2$, represents a circle, find its centre and radius.

4. Examine whether the function $f(z)=\bar{z}$ is analytic or not.

5. Prove that $\oint_C \dfrac{z}{z-2}dz = 0$, if C is the circle $|z|=1$

6. Find the fixed point of transformation $w=\dfrac{2z-5}{z+4}$

7. Evaluate $\int_C (3x+y)\,dx+(2y-x)\,dy$ along the curve $y=x^2+1$ from $(0,1)$ to $(3,10)$.

8. Evaluate $\int_0^4 \int_0^{\sqrt{y}} xy\,dx\,dy$

9. Evaluate $\int_0^a \int_0^a \int_0^a (yz+zx+xy)\,dx\,dy\,dz$

10. Prove that $\beta(m,n)=\beta(n,m)$

11. Evaluate $\int_0^1 x^{3/2}(1-x)^{1/2}\,dx$

12. Evaluate $\frac{\Gamma(3)\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{11}{2}\right)}$

II. ***Answer any THREE of the following:*** 3X5=15 Marks

1. Show that $Arg\left(\frac{z-1}{z+2}\right)=\frac{\pi}{4}$ represent a circle. Find its centre and radius.
2. Show that the function $u(x,y)=x^3-3xy^2$ is harmonic and find the harmonic conjugate also write the corresponding analytic function.
3. If $f(z)=u+iv$ is analytic then prove that u and v satisfy C – R equations.
4. Prove that an analytic function with constant modulus is constant.
5. If $f(z)=u+iv$ is analytic and u and v are harmonic functions then show that

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$$

III. ***Answer any THREEof the following:*** 3x5=15 Marks

1. State and prove Cauchy's Integral formula.
2. Evaluate $\oint_C z^2 dz$, where C is
   i) The line segment joining the points 0 to $3+i$
   ii) The path consisting of two line segments are $z=3+i$ and $z=3$.
3. Evaluate $\oint_C \frac{3z-1}{z^3-z}dz$, where C is the circle.
   i) $|z|=\frac{1}{2}$
   ii) $|z|=2$
4. Find the bilinear transformation which maps $z=(i,-i,1)$ onto $w=(0,1,\infty)$
5. Discuss the transformation $w=z^2$ when real past of z is constant.

Contd.........3

II. *Answer any THREE of the following:* 3 X 5 = 15 Marks

1. Find the equation of the circle passing through the points $1, i, 1+i$. Alsofind its centre and radius.
2. Show that $f(z) = \cosh z$ is analytic and $f'(z) = \sinh z$.
3. State and prove the necessary condition for a function $f(z)$ to be analytic in a domain D.
4. Show that $u = \cos x \cosh y$ is harmonic and find the analytic function whose real part is u.
5. Find the orthogonal trajectory of family of curves $x^2 - y^2 + x = e$, where e is a constant.

III. *Answer any THREE of the following:* 3 x 5 = 15 Marks

1. Evaluate $\int_0^{3+i} z^2 2z$ along $3y = x$.
2. State and prove Cauchy's integral theorem.
3. Evaluate $\int_C \frac{\left[\sin(\pi z^2) + \cos(\pi z^2)\right] dz}{(z-1)(z-2)}$ where 'C' is the circle $|z| = 3$
4. Show that the transformation $w = \frac{1}{z}$ transforms a circle into a circle or a straight line.
5. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = 0, i, \infty$

IV. *Answerany THREE of the following:* 3 x 5 = 15 Marks

1. If f is a vector function of t then show that $\frac{d}{dt}\left[f \; \frac{df}{dt} \; \frac{d^2 f}{dt^2}\right] = \left[f \; \frac{df}{dt} \; \frac{d^3 f}{dt^3}\right]$
2. For the curve $k = a\cos t, y = a\sin t, z = bt$ show that $K = \frac{a}{a^2 + b^2}$, $J = \frac{b}{a^2 + b^2}$
3. Find the unit normal vector and equation of the tangent plane to the surface $\vec{r} = (u+v)\hat{i} + (u-v)\hat{j} + (4u^2)\hat{k}$ at $u = 1, v = 2$
4. Express the vector $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in terms of cylindrical coordinates.
5. Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{x} = \frac{y-2}{-3} = \frac{z-3}{5}$.

Contd.........6

Mathematics

QP CODE 15632

3 x 5 = 15 Marks

V. *Answer* **any** ***THREE of the following:***

1. Let f be a bounded function on $[a,b]$. If P^* is a refinement of the partition P of $[a,b]$ then prove that $L(p,f) \le L(p^*,f)$

2. Prove that a function f is Riemann integrable on $[a,b]$ if and only if for every $\in > 0$ there exists a partition P such that $U(P,f) - L(P,f) < \in$

3. Prove that every monotonic function is Riemann Integrable.

4. Show that $\int_1^2 f(x)\,dx = \frac{11}{2}$ where $f(x) = 3x+1$ using the integral as the limit of a sum.

5. Show that the function $f(x) = \begin{cases} \frac{1}{2^n} & if \quad \frac{1}{2^{n+1}} < x < \frac{1}{2^n} \\ & \quad \text{for } x = 1,43 \\ 0 & if \quad x = 0 \end{cases}$ is integrable in $[0,1]$ and $\int_5^1 f(x)\,dx = \frac{2}{3}$.

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Sixth Semester B.Sc. Degree Examinations

APRIL /MAY 2022

(Semester Scheme)

(Before 2014 – 15 Old Syllabus)

MATHEMATICS

SSF 531: Paper – VIII

Time: 3hrs.] [Max. Marks: 80

Note: Answer ALL questions.

I. ***Answer any TEN of the following.*** 10 x 2 = 20 Marks

1. Find the real and imaginary parts of $\sin z$
2. Evaluate $\lim_{Z\to i}\left(\frac{Z^2+1}{Z^6+1}\right)$
3. Show that the function $f(z)=\overline{z}$ is not analytic at any point.
4. Show that $u=e^x\cos y+xy$ is harmonic.
5. Evaluate $\int_C \frac{e^z dz}{z^2}$ where C is $|z|=1$.
6. Find the fixed points of $w=\frac{3z-4}{z}$
7. If $\vec{A}=t^2\hat{i}+t\,\hat{j}-t^3\,\hat{k}$ and $\vec{B}=t^3\hat{i}+2t^4\,\hat{j}+t^2\,\hat{k}$ Find $\frac{d}{dt}\left(\vec{A}.\vec{B}\right)$ at $t=1$.
8. Find the unit tangent vector to the curve $x=2t,\ y=t^2,\ z=\frac{t^3}{3}$, at $t=1$
9. Find the cylindrical coordinates of the point whose Cartesian coordinates are $\left(1,\sqrt{3},3\right)$
10. Define upper and lower integrals of a bounded function on $[a,b]$.
11. Corresponding to any partition P of $[a,b]$ show that $L(p,f)\le U(p,f)$
12. State the Fundamental theorem of Integral calculus.

Contd.........5

IV. ***Answerany THREE of the following:*** 3 x 5 = 15 Marks

1. Evaluate $\int_C (2+x^2 y)\,ds$, where C is the upper half of the unit circle $x^2+y^2=1$.

2. Evaluate $\int_C (x^2-y)\,dx+(y^2+x)\,dy$, where C is the curve given by $x=t, y=t^2+1$ where $0 \le t \le 1$.

3. Evaluate $\iint_R y\,dx\,dy$ over the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

4. Evaluate by changing the order of integration $\int_1^2 \int_1^{x^2} (x^2+y^2)\,dy\,dx$.

5. Find the surface area of the sphere of radius $'a'$.

V. ***Answer* any *THREE of the following:*** 3 x 5 = 15 Marks

1. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$

2. Evaluate $\iiint_V (x^2+y^2+z^2)\,dx\,dy\,dz$ where V is bounded by $x+y+z=a$ and $x=0, y=0, z=0$

3. Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $m, n>0$

4. Evaluate $\int_0^4 x^{3/2}(4-x)^{5/2}\,dx$

5. Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}.\ d\theta$ by expressing in terms of gamma functions.

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Contd.........4